Introduction to Particle Physics I

units and interactions

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Spring 2015
Lecture I
units and elementary interactions
outline

• Lecture I: Units, Elementary Interactions
• Lecture II: Relativistic kinematics
• Lectures III: Lorentz invariant scattering cross section
• Lecture IV: Accelerators and collider experiments
• Lecture V: Elements of Quantum Electrodynamics - QED
• Lecture VI: Testing QED
• Lecture VII: Unitary symmetries and Quantum ChromoDynamics – QCD - as a gauge theory
• Lectures VIII: QCD in $e^+e^-$ annihilation
Lecture I; Orientation, Units & Elementary Interactions

References:

- Halzen & Martin [I.1]
- Aitchison & Hey [I.2]
- Seiden [I.3]
- Nachtmann [I.4]
Particle Physics deals with the basic concepts of matter and energy – What constitutes matter and/or energy?

Elementary matter/energy entities with a set of quantum properties - no resolved structure. Their interactions with the known environment are usually well defined and are being used to observe the entities:

• gravity (?)
• electromagnetic interaction
• weak interaction
• strong interaction
the concept of basic matter...

<table>
<thead>
<tr>
<th>YEAR</th>
<th>WHO/WHERE</th>
<th>WHAT</th>
<th>CONCEPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1869</td>
<td>Mendeleev, Meyer</td>
<td>periodic system</td>
<td>atom</td>
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<tr>
<td>1890</td>
<td>J.Thomson</td>
<td>electron</td>
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<tr>
<td>1910</td>
<td>Bequerel, Curie, Rutherford</td>
<td>radioactivity</td>
<td>atomic nucleus</td>
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<tr>
<td>1932</td>
<td>Chadwick, Anderson</td>
<td>neutron/positron</td>
<td>$p,n,e^-/\bar{p},\bar{n},e^+$</td>
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<td>1947</td>
<td>Blackett, Powell</td>
<td>pion/muon</td>
<td>particle zoo...</td>
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<td>1956</td>
<td>Cowan, Reines</td>
<td>neutrino</td>
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<tr>
<td>1967</td>
<td>Glashow, Weinberg, Salam</td>
<td>EW theory</td>
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<tr>
<td>1968</td>
<td>SLAC</td>
<td>DIS</td>
<td>quarks &amp; leptons</td>
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<tr>
<td>1972</td>
<td>Fritzsch, Gell-Man, Leutweyler</td>
<td>QCD</td>
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<td>1974</td>
<td>SLAC&amp;BNL</td>
<td>c- quark/τ-lepton</td>
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<td>1979</td>
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<td>1977</td>
<td>Fermilab</td>
<td>b-quark</td>
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<td>1983</td>
<td>CERN</td>
<td>W- &amp; Z-bosons</td>
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<tr>
<td>1995</td>
<td>Fermilab</td>
<td>t-quark/$\nu_\tau$</td>
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<tr>
<td>2013</td>
<td>CERN</td>
<td>Higgs!</td>
<td>concept of mass...</td>
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</table>
phenomena

- Instability of matter – radioactivity – decay of elementary particles

- Scattering of elementary particles

- Production of new particles

- Indirect implications:
  - early history of the universe
  - fuel cycle in stars
  - astrophysics: supernovae, very high energy cosmic rays


four-vector notation and invariants

four-vector notation:
the energy and momentum of a particle making its relativistic properties apparent

• definition: objects which transform like \((ct, x, y, z)\) under Lorentz transformations (include boosts and rotations)

• length of a four-vector: an invariant – does not change under Lorentz transformations; the scalar product of any pair of four-vectors is an invariant; length of four vectors can attain values that are negative or zero.
energy-momentum

- energy-momentum four vector

\[ p^\mu = (p^0, p^1, p^2, p^3) = \left( \frac{E}{c}, p_x, p_y, p_z \right) = \left( \frac{E}{c}, \vec{p} \right) \]

- where E is the energy of the particle and \(p_x\), \(p_y\) and \(p_z\) are the components of particle momentum.

\[ p^\mu \cdot p_\mu = \frac{E^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = m^2 c^2 \]

- length of a four-vector is an invariant – it does not change under Lorentz transformation
natural units

• time and space are closely related – they mix up under Lorentz boosts – linkage through speed of light (dimensions \((LT^{-1})\))

• in Quantum Mechanics: energy & time (momentum&distance)

• by choosing: \(\hbar = c = 1\) can express everything in terms of powers of energy, since

\[
M = \frac{E}{c^2}, \quad L = \frac{\hbar c}{E}, \quad T = \frac{\hbar}{E}
\]
natural units

- The Planck constant \( \hbar = \frac{\hbar}{2\pi} = 1.0546 \cdot 10^{-34} \text{ Js} \)
- The speed of light \( c = 2.998 \cdot 10^8 \frac{\text{m}}{\text{s}} \)
- The ”natural units”: \( \hbar = c = 1 \)

\[
[c] = [\text{length}] \cdot [\text{time}]^{-1} = [L][T]^{-1} \Rightarrow [L] = [T]
\]

\[
[\hbar] = [\text{energy}] \cdot [\text{time}] = [M][L]^2[T]^{-1} \Rightarrow [M] = [L]^{-1}
\]

\[
\Rightarrow [M] = [L]^{-1} = [T]^{-1} \text{ and } [E] = [M]
\]
unit for energy

\[ 1 \text{eV} = 1.602 \cdot 10^{-19} \text{J} \]
\[ k\text{eV} = 10^3 \text{eV} \]
\[ M\text{eV} = 10^6 \text{eV} \]
\[ G\text{eV} = 10^9 \text{eV} \]
\[ T\text{eV} = 10^{12} \text{eV} \]
\[ P\text{eV} = 10^{15} \text{eV} \]

\[ m_e = 511 \text{keV} \]
\[ m_p = 938 \text{MeV} \]
\[ m_n = 939 \text{MeV} \]

\[ E_e (\text{LEP}) = 104.5 \text{GeV} \]
\[ E_p (\text{Tevatron}) = 980 \text{GeV} \]
\[ E_p (\text{LHC}) = 7 \text{TeV} \]
\[ E_p (\text{VHE – cosmic rays}) = 1 \text{PeV} \]
energy scales - examples

- **230 µeV**: the thermal energy $k_B T$ of the cosmic microwaves
- **25 meV**: the thermal energy $k_B T$ at room temperature; one air molecule has an average kinetic energy 38 meV
- **1.6 eV to 3.4 eV**: the photon energy of visible light
- **13.6 eV**: the energy required to ionize atomic hydrogen; molecular bond energies are on the order of 1 eV to 10 eV per bond
- **1 MeV**: megaeV ($1.602 \times 10^{-13}$ J): about twice the rest energy of an electron
- **17.6 MeV**: the average energy released in the fusion of deuterium and tritium to form He-4; this is 0.41 PJ per kilogram of product produced
- **200 MeV**: the average energy released in nuclear fission of one U-235 atom
- **210 MeV**: the average energy released in fission of one Pu-239 atom
energy scales – examples...

- **1 TeV**: a tera electronvolts, or $1.602 \times 10^{-7} \, \text{J}$, about the kinetic energy of a flying mosquito$^9$
- **14 TeV**: the designed proton collision energy at the Large Hadron Collider (which has operated at half of this energy since 30 March 2010)
- **1 PeV**: one petaelectronvolt, the amount of energy measured in each of two different cosmic neutrino candidates detected by the IceCube neutrino telescope in Antarctica$^8$
- **300 EeV**: exaeV ($3 \times 10^{20} \, \text{eV} = \sim 50 \, \text{J}$):$^7$ the so-called Oh-My-God particle (the most energetic cosmic ray particle ever observed)
- **~0.624 ZeV**: zettaeV ($6.24 \times 10^{20} \, \text{eV}$): energy consumed by a single 100-watt light bulb in one second ($100 \, \text{W} = 100 \, \text{J/s} \approx 6.24 \times 10^{20} \, \text{eV/s}$)
- **2435 YeV**: yottaeV ($2.435 \times 10^{27} \, \text{eV} \approx 4.341 \times 10^{-9} \, \text{kg} =$ nature’s maximum allowed mass for point-masses
- **5.25\times10^8 \text{ YeV} = 5.25 \times 10^{32} \, \text{eV}**: total energy released from a 20 kt nuclear fission device
- **125.3\pm0.6 \text{ GeV}**: the energy emitted by the decay of the Higgs Boson, as measured by two separate detectors at the LHC to a certainty of 5 sigma$^{10}$
unit conversions

\[ \hbar = 6.58 \cdot 10^{-25} \text{GeV} \cdot \text{s}^{-1} \equiv 1 \implies 1 \text{GeV}^{-1} \approx 6.58 \cdot 10^{-25} \text{s} \]

Particle life time \( \tau = \frac{1}{\Gamma} \), where \( \Gamma \) is the width of a resonance, and

\[ c = 2.998 \cdot 10^8 \frac{m}{\text{s}} \equiv 1 \implies 1 \text{fm} = 10^{-15} m \approx \frac{1}{200 \text{MeV}} \]
unit conversions

Cross sections have dimensions of area:

\[ [\sigma] = [L]^2 = [M]^2 = \frac{1}{(eV)^2} \]

As the unit, we choose

\[ \frac{1}{(GeV)^2} = 389379\text{nb} = 389379 \cdot 10^{-9} b \]

with 1b : 1barn = $10^{-24}$ cm$^{-2}$ the typical scale of nuclear absorption
the unit of electrical charge

\[ \alpha = \left. \frac{e^2}{4\pi\varepsilon_0\hbar c} \right|_{SI} = 7.2972 \cdot 10^{-3} \approx \frac{1}{137} \]

\[ = \left. \frac{e^2}{\hbar c} \right|_{CGS} \]

\[ = \left. \frac{e^2}{4\pi\hbar c} \right|_{Heaviside-Lorenz} \]

Therefore, in Heaviside-Lorentz units, the electron charge is fixed to be

\[ e = \sqrt{4\pi\alpha} \left|_{Heaviside-Lorenz} \right. \]
elementary interactions
Gravitation, Electromagnetic, Weak and Strong Interactions
Gravitation and electrostatics

Newton’s law:
Force of gravitation between two bodies 1 and 2

\[ F = \Gamma \cdot \frac{m_1 \cdot m_2}{R^2} \]

Coulomb’s law:
Electrical force between two bodies 1 and 2 carrying electrical charge \( q_i \)

\[ F = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q_1 \cdot q_2}{R^2} \]

Same formula, but different constants!
Gravitation and electrostatics - bound state of two particles

Earth-Moon

Hydrogen atom - same formula, different constants

\[ F = \Gamma \cdot \frac{m_1 \cdot m_2}{R^2} \]
Gravitation and electrostatics
- compare gravitation & em force

interaction strength between two bodies with equal masses $m_P$ and electrical charges $e$ are equal, if

$$\Gamma \cdot \frac{m_P \cdot m_P}{R^2} = \frac{1}{4\pi \varepsilon_0} \cdot \frac{e \cdot e}{R^2} \Rightarrow m_P = 21.77 \mu g \text{ (Planck mass)}$$

• electrical charges appear in whole numbers of $e$
• E: attractive or repulsive, G: always attractive?
• G: $m_T = m_S$ mass of inertia=mass of gravity, all bodies feel the same acceleration (Galilei’s law of free fall)
Electrodynamics - magnetostatic forces

ancient experimental evidence

complicated formulation proportional to $R^3$
includes torque
Electrodynamics
-unification of electrical
-and magnetic forces

Formulated by Faraday & Maxwell in 1800’s;

• Magnetic fields created by moving electrical charges – time-varying electrical fields
• Time-varying magnetic fields create electrical fields
• A set of 4 Maxwell Equations describe all electro-magnetic effects consistently
• Electrical and magnetic forces represent different phenomena – have their origins in the same basic physics law
• Only one constant: c
• Magnetic monopoles?
Electrodynamics - explains new effects

Maxwell equation describes behaviour of electromagnetic waves - such as light

Speed of light is constant = c; independent of speed of source or speed of the observer

⇒ theory of special relativity
   (Albert Einstein 1905)

Maxwell's electrodynamics is probably the best theory we have so far:
• it truly unifies two forces
• it explains additional effects, like light
• it predicts correctly the theory of special relativity.
Strong Force
- confinement of colour charge

\[ E_{pot} \]

keeps rising!
Weak Force
- what makes the sun shining
hydrogen atoms melt together into He-4 (pn->ppnn)

exchange some of the protons into neutrons; u-quarks into d-quarks
⇒ β-decay through weak interaction
Weak and Strong Force
- properties of the weak interaction

responsible for the $\beta$-decay

has a very short range:

$$E_{pot} \approx -\frac{1}{R} e^{-\mu R}$$

with $\approx 1/0.002 \, fm$

the only fundamental interaction with this short range

at very small distances has the same behaviour as other interactions – but different constant term
The Four Fundamental Interactions
- classical description

Each interaction acts with its own strength
- how about the small-R behaviour?

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity</td>
<td>infinite</td>
</tr>
<tr>
<td>Electrical</td>
<td>infinite</td>
</tr>
<tr>
<td>Weak</td>
<td>short</td>
</tr>
<tr>
<td>Strong</td>
<td>confinement</td>
</tr>
</tbody>
</table>

$E_{pot}$ vs $R$ graph
How to measure?
- the scattering experiment

beam energy determines $R_{\text{min}}$
- resolution

measure the energies and polar angles ($\theta$) of the scattered particles
The scattering experiment - determine the law of interaction

1/sin^4(\theta/2)
The scattering experiment - what more to learn?

- from over-all scattering probability $\rightarrow$ coupling constant
- from energy of the scattered particle $\rightarrow$ momentum of the target particle
- energy of the beam particle determines the distance of approach: $R_{\text{min}}$ $\rightarrow$ ”resolution” of the experiment - need high energies for finding the systematics at small distances
- usually many particles produced $\rightarrow$ gain more details on the scattering process/type of interaction
Quantum mechanics - brings in modifications

- discrete energy and angular momentum states in bound systems – $\text{H}_2$ atom
- interaction field comes in quanta, as well

“interaction carriers” – “exchange particles” – “propagators” are all particles/quanta
### Quantum Field Theory
- **field quanta of the fundamental interactions**

The standard model of particle physics is a QFT and describes:

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Quanta</th>
<th>Mass</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetism</td>
<td>Photon</td>
<td>$\text{mass}=0$</td>
<td>Electrical</td>
</tr>
<tr>
<td>Weak interaction</td>
<td>W/Z</td>
<td>$\approx90\text{GeV}$</td>
<td>Weak</td>
</tr>
<tr>
<td>Hypercharges</td>
<td>Gluon(s)</td>
<td>$\text{mass}=0$</td>
<td>Colour</td>
</tr>
</tbody>
</table>

The propagators transfer energy and momentum. Each interaction has its own coupling constant.
Electron-proton scattering
- DESY ep collider - HERA
Electron-proton scattering
- ep collider DESY
Electron-proton scattering
- ep collider DESY
Unification of forces?

- coupling strengths vary slowly with energy
- theoretically predicted

Present experiments:

- strong
- weak
- electromagnetic

Energy $\approx 1/R_{\text{min}}$
Strong coupling evolves with energy

**HERA**

- **ZEUS**
- **H1**

QCD

\[ \alpha_s(M_Z) = 0.118 \pm 0.003 \]

Energy, \( \sim 1/R \)
summary

**long distance (low energy):**
forces described by their potential energy – 4 known fundamental forces: gravity, electromagnetic, strong and weak

**short distance (high energy):**
forces described by Quantum Field Theory (QFT) – exchange particles

**the standard model (SM)** of particle physics is a QFT that describes three different interactions: electromagnetic & weak (‘electroweak’) and strong

Are the three SM forces different, low energy manifestations of the same fundamental interaction? Can they be unified at higher energies?
gravitational interaction

Since

\[ Gm_p^2 \approx 10^{-39} \]

and because the range of gravitational interaction is infinite, gravitation is not relevant here.

\textbf{Note, however, that gravitation should be part of the unified theory!}
electromagnetic interaction

\[ \alpha = \frac{1}{137} \]

The range of the em interaction is infinite and typical lifetimes of particles that decay through em interactions range from

\[ \tau_{\Sigma^0 \rightarrow \Lambda^0 \gamma} = 10^{-20} \text{ s} \text{ to } \tau_{\pi^0 \rightarrow \gamma \gamma} = 10^{-16} \text{ s}. \]

Typical cross sections are of the order of

\[ \sigma_{e p \rightarrow e p} = 1 \mu b. \]
electromagnetic interaction

QED predictions are tested to extreme precision – both in theory and experiment. Consider the anomalous magnetic moment of the electron:

\[
\frac{\mu_e}{2m_e} = \frac{e}{2m_e} \left\{ \begin{array}{c}
1 \text{ Dirac} \\
\frac{1}{2} \text{ \frac{\alpha}{\pi}} \text{ Schwingen} \\
-0.388 \text{ \frac{\alpha^2}{\pi^2}} \\
1.18 \text{ \frac{\alpha^3}{\pi^3}}
\end{array} \right\} \\
= \frac{e}{2m_e} \{1.0022596521465(270)\}
\]

\[
\frac{\mu_e}{2m_e} \{1.0011596521883(42)\}
\]

by Van Dyck, Schwinberg and Dehmelt
weak interaction

Consider $\beta$ decay: $n \rightarrow p e \bar{v}_e$

\[ G_F m_p^2 \approx 10^{-5} \]

range $\sim$ 1 fm, lifetimes from $10^{-10}$ to $10^3$ s, cross sections $\sigma \sim 1$ fb
strong interaction

Yukawa theory \[ m_\pi = 130\text{MeV} \Rightarrow \text{range} \approx \frac{1}{m_\pi} = \frac{1}{130\text{MeV}} \approx 1.4\text{fm} \]
strong interaction potential

\[ V(r) \approx \frac{1}{r} \approx 1 \text{ fm} \]

\[ \alpha_s \approx 0.12 \]
**Feynman diagrams**

**VERTICES**

- at vertices, take care of all relevant conservation laws (4-momentum & quantum numbers – ‘conserved currents’)

- each vertex introduces a dimensionless coupling strength $\sqrt{\alpha}$ which is less than 1, and is defined for each type of interaction; therefore, diagrams with the least number of vertices contribute – in general – most to the process

**CROSSING**

- relate apparently disconnected processes by switching the space and time directions
NEXT: Lecture II; Relativistic Kinematics

• Particle decay

• Two-particle scattering
  • Scattering angle
  • Elastic scattering
  • Angular distribution
  • Relative velocity
  • Center of mass and laboratory systems

• Crossing symmetry
  • Interpretation of antiparticle-states