Introduction to Particle Physics I

weak interactions

Risto Orava
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lecture

weak interaction
the standard model
-weak interactions

- examples of weak interactions
- properties of weak interaction
- weak interaction and quarks
- cross section examples
- quark mixing
- lepton mixing
- neutral weak interactions

for reference see Halzen&Martin
page 21 and pages 251-254
weak interaction properties

• properties of the weak interaction
  – exchange of W and Z bosons

• does not conserve the same quantities
  – flavour *not* conserved
  – *parity broken*
  – *charge parity broken*

• involves all fermions including neutrinos
  – hint: if neutrinos involved it must be weak.

• cross section: \( \sigma \approx 10^{-38} \text{m}^2 \)
  – cf. size of the proton \( R \approx 10^{-15} \text{m} \)
weak interactions...

• if weak interaction involves the exchange of a $Z$ boson then

$$\Delta Q = 0$$

• if weak interaction involves exchange of $W$ boson then the charge must change by 1

$$\Delta Q = \pm 1$$

• if strangeness is not conserved we get:

$$\Delta S = \Delta Q = \pm 1$$
weak interactions...

• neither the em nor strong interactions change flavour
• if these were the only forces, the number of $e^+e^-/\mu^+\mu^-/\tau^+\tau^-/u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}/b\bar{b}/t\bar{t}$-pairs in the Universe would be a constant
• the weak interactions change flavour, there are charged and neutral weak interactions
• consider first charged weak interactions; for leptons, the basic interaction is:
weak interactions...

• $e^-$ always couples to the $\nu_e$, $\mu$- to the $\nu_\mu$, etc.

• equivalent to *conservation* of electron, muon and tau number

• the weak charge ("*weak isotopic spin*" is also used): in the sense that the EM interaction has one charge and the strong interaction has three charges, the weak interactions have **two** charges

• like the gluon (and unlike the photon) the W-boson carries weak charge

• the W is very *massive*, i.e. it is extremely virtual at low energies - its interactions at low energies are, therefore, very weak and screening effects are minor

• **all the charged weak charges are equal to each other**
weak interactions...

• by combining two fundamental vertices, we can describe the $\mu$ decay:

\[
\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu
\]

• the fundamental quark charged weak interactions are approximately:

Note: The charges are equal to the lepton couplings.
examples weak interaction

\[ \nu_e + p \rightarrow e^- + \pi^+ + p \]

\[ \nu_\mu + e^- \rightarrow \nu_\mu + e^- \]

\[ \Omega^- \rightarrow \Xi^- + \pi^+ + \pi^- \]
weak interactions - examples

Decays

\( n \to p + e^- + \bar{\nu}_e \) \( (\tau = 889 \text{ s}) \)

\( \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \) \( (\tau = 2.2 \times 10^{-6} \text{ s}) \)

\( \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \)

\( \pi^+ \to \mu^+ + \bar{\nu}_\mu \) \( (\tau = 2.2 \times 10^{-8} \text{ s}) \)

\( \tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau \) \( (\tau = 2.2 \times 10^{-13} \text{ s}) \)

\( \to e^- + \bar{\nu}_e + \nu_\tau \)
weak interactions - examples

\[ \bar{\nu}_e + p \rightarrow n + e^+ \quad \text{(allowed)} \]

\[ \nu_e + \bar{p} \rightarrow \bar{n} + e^- \quad \text{(allowed)} \]

\[ \nu_e + p \rightarrow n + e^+ \quad \text{(forbidden, lepton number)} \]

\[ \nu_e + p \rightarrow p + \pi^+ + e^- \quad \text{(allowed)} \]

\[ \nu_e + e^- \rightarrow \nu_e + e^- \quad \text{(allowed)} \]

\[ \nu_e + n \rightarrow \nu_e + n \quad \text{(allowed)} \]
\[ \pi^- \rightarrow \mu^- \nu_\mu \]

\[ \pi^- \rightarrow \mu^- \bar{\nu}_\mu \]
\[ \pi^- \rightarrow e^- \bar{\nu}_e \]

• this is not the whole story - otherwise the K\(^-\) would be stable:

\[ K^- \]

c is heavier than s, and K\(^-\) cannot decay that way – suppressed decays..
\[ K^- \rightarrow \mu^- \nu_\mu \]

• the \( K^- \) does decay in the same way as the \( \pi^- \), but with a decay width which is \textbf{20 times smaller} - after correcting for the mass difference.

• the solution to this puzzle was first suggested by Nicola Cabibbo in 1963, and then extended by Glashow, Iliopoulos and Maiani in 1970 when they suggested the \textit{charmed} quark.

• the weak charge is basically universal, but the weak eigenstates are \textit{rotated} from the strong eigenstates.
weak interaction and quarks

- W and Z bosons **do not** couple to the same quark states as the strong and electromagnetic interaction does.
- The quark states that **do couple** to the strong and EM interactions are called the **mass eigenstates**.
- The weak interaction couples with the weak eigenstates.

- Note: \( u = u' \)
- \( c = c' \)
- \( d' \) and \( s' \) are **not**
- \( d \) and \( s \)

\[
\begin{align*}
\text{Leptons:} & \begin{pmatrix}
 e \\
 \nu_e
\end{pmatrix} \quad \begin{pmatrix}
 \mu \\
 \nu_\mu
\end{pmatrix} \\
\text{Quarks:} & \begin{pmatrix}
 u \\
 d'
\end{pmatrix} \quad \begin{pmatrix}
 c \\
 s'
\end{pmatrix}
\end{align*}
\]
weak eigenstates of $d'$ and $s'$

- the weak eigenstates are combinations of the mass eigenstates $d$ and $s$.
- only $u$, $d$, $c$, and $s$ considered for now.
- write them as:

$$
\begin{align*}
  d' &= d \cos \theta_C + s \sin \theta_C \\
  s' &= -d \sin \theta_C + s \cos \theta_C \\
\end{align*}
$$

where $\theta_C = \text{Cabbibo Mixing Angle}$
rates of $\pi$ and $K$ decays

- measure the Cabbibo angle by using the $\pi$ and $K$ decays:

\[
\tau(K^-) \approx \frac{\Gamma(K^- \rightarrow \mu^- + \bar{\nu}_\mu)}{\Gamma(\pi^- \rightarrow \mu^- + \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2 \theta_C
\]

\[
K^- \rightarrow \mu^- + \bar{\nu}_\mu
\]

\[
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu
\]
The standard interaction can be broken into two components, one involving down and the other strange eigenstates:

\[ g_{ud} = g_W \cos \theta_C \quad g_{us} = g_W \sin \theta_C \]
decays of the charm quark

• the charm quark can decay weakly via two different quark paths:

\[ c \rightarrow s + l^+ + \nu_l \]

\[ c \rightarrow d + l^+ + \nu_l \]

• the ratio of the rates of these two processes is given by:

\[ \frac{g_{cs}^2}{g_{cd}^2} = \frac{\cos^2 \theta_C}{\sin^2 \theta_C} \approx \frac{1}{20} \]
visualization of the eigenstates

• mixing of the mass eigenstates form the weak eigenstates through a rotation matrix:

\[
\begin{pmatrix}
  d' \\
  s'
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta_C & \sin \theta_C \\
  -\sin \theta_C & \cos \theta_C
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix}
\]
extension to all quark flavours

• extend the rotation matrix to all three known families of quarks using a 3 by 3 rotation matrix:

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\approx
\begin{pmatrix}
  \cos \theta_C & \sin \theta_C & 0 \\
  -\sin \theta_C & \cos \theta_C & 0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]
quark mixing

• extension to 3 families: need 4 free parameters, 3 real numbers are needed to describe the rotation angles, and there is 1 irreducible phase factor

• the 3x3 matrix is called the *Cabibbo-Kobayashi-Maskawa* matrix

• a number of parametrizations exist for the CKM matrix, the one by *Lincoln Wolfenstein* is the most popular one:

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  1 - \frac{\lambda^2}{2} & \lambda & \lambda^3 A(\rho - i\eta) \\
  -1 & 1 - \frac{\lambda^2}{2} & \lambda^2 A \\
  \lambda^3 A(1 - \rho - i\eta) & -\lambda^2 A & 1
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]
quark mixing

- the mixing matrix is of 3rd order in $\lambda$, the 4 parameters are: $\lambda$, $\Lambda$, $\rho$, and $\eta$ (the complex mixing is represented by $\eta$)
- the $U_{23}$ element is chosen to be $\approx \lambda^2$ following the experimental data which show that the transition $b \rightarrow c$ is suppressed relative to the $s \rightarrow u$
- magnitudes of the CKM matrix (see the PDG data for updates!)

\[
\begin{bmatrix}
0.9745 \pm 0.0006 & 0.224 \pm 0.003 & 0.0029 - 0.0045 \\
0.224 \pm 0.003 & 0.9737 \pm 0.0006 & 0.039 - 0.044 \\
0.0048 - 0.014 & 0.040 \pm 0.003 & 0.9991 \pm 0.0001
\end{bmatrix}
\]

- the errors are given at 90% C.L.

Note: The 3rd family quarks mix very weakly, and the 2x2 Cabibbo matrix is sufficient if a process has the 1st and 2nd family quarks.
\[ \tau \rightarrow e^- \bar{\nu}_e \nu_\tau \]

- an important branching ratio:

\[ B(\tau \rightarrow e^- \bar{\nu}_e \nu_\tau) \equiv \frac{\Gamma(\tau \rightarrow e^- \bar{\nu}_e \nu_\tau)}{\Gamma(\tau \rightarrow all)} \]

- possible decay modes are:

\[ B(\tau \rightarrow e^- \nu_e \nu_\tau) = 1/5 = 20\%^* \]

* The measured value is 18%. The difference can be attributed to the higher order effects.
lepton mixing?

- how about lepton mixing? is the lepton matrix also rotated?
  - probably yes – due to the very tiny masses of the neutrinos (see PDG listings!), the mixing is only visible through neutrino oscillations.

- consider only two flavours, e and μ - assume then two mass eigenstates: \( \nu_1 \) and \( \nu_2 \) such that:
  
  \[
  \nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta \\
  \nu_\mu = \nu_2 \cos \theta - \nu_1 \sin \theta
  \]

  so that \( \langle \nu_e | \nu_\mu \rangle = 0 \)

- by \( \nu_\mu \), it is meant that the state is prepared as in π decay:
  \[ \pi^- \rightarrow \mu^- \nu_\mu \]

- according to QM, a state develops in time as:
  
  \[
  |t\rangle = e^{-iHt/\hbar}|t = 0\rangle
  \]
lepton mixing...?

• in the rest frame of the $\nu$, $E=mc^2$ AND

$$\left|\nu_\mu(t)\right\rangle = \exp(-im_2 c^2 t / \hbar) \cos \Theta \left|\nu_2\right\rangle - \exp(-im_1 c^2 t / \hbar) \sin \Theta \left|\nu_1\right\rangle$$

• if $m_1 = m_2$, then

$$\left|\nu_\mu(t)\right\rangle = \exp(-im c^2 t / \hbar)(\cos \Theta \left|\nu_2\right\rangle - \sin \Theta \left|\nu_1\right\rangle)$$

$$= \exp(-im c^2 t / \hbar) \left|\nu_\mu\right\rangle$$

• the phase coherence of the $\nu_\mu$ never changes, and it is always orthogonal to $\nu_e$, if $m_1 \neq m_2$, then for $t > 0$, $<\nu_e | \nu_\mu > \neq 0$, and $\nu_\mu$ will oscillate into $\nu_e$.

• for $m_\nu$ small, this takes a long time and is difficult to observe

• $\nu_\mu$ appears to oscillate into something, probably into $\nu_\tau$, over distances between 20km and 10,000km from $\nu$'s produced by the earth's atmosphere.

• there is also evidence that $\nu_e$ produced in the sun oscillate into something, possibly into $\nu_\mu$, by the time they reach the earth.
neutral weak interactions

- consider reaction $K^- \rightarrow \pi^- \bar{\nu} \bar{\nu}$:

- this reaction has **not** been observed within the upper limit of: $B(K^- \rightarrow \pi^- \nu \bar{\nu}) < 2.4 \times 10^{-9}$ - this despite of the fact that the decay:

  - has a measured branching ratio of $3.2 \times 10^{-2}$. 
neutral weak interactions

- *neutral* weak interactions – *neutral currents* – were not expected to exist – however, in 1973 they were discovered by a bubble chamber group at CERN in reactions of the type:

- the $Z^0$ decays preserve flavour if every $q=-1/3$ quark is balanced by a $q=2/3$ quark

- in 1970 Glashow, Iliopoulos and Maiani ’predicted’ the charmed quark to exist

- the $Z^0$ acts like a heavy photon - its effects are usually masked by the much stronger EM interactions - an exception is when $\nu$'s are involved - they have no EM interactions!
weak interactions...

- both W and Z carry weak charges - they interact with each other via the following fundamental vertices:

Note: Up to now these diagrams have not been important - with the Tevatron, LHC and the Next Linear Collider, these processes will be seen...
conservation laws

• all the particle decays allowed by the fundamental interactions, will take place if they are *kinematically allowed* – provided there is no *symmetry principle* forbidding the process
• for example: the decay $\rho^0 \rightarrow \pi^0 \pi^0$ is forbidden as a strong decay, but there is no way of knowing this yet...

=> for any possible decay mode, first check whether it is allowed kinematically

• there are absolute conservation laws which are obeyed by *all* interactions:
  (1) *electric charge*
  (2) *baryon number*
  (3) *electron, muon, and tau (lepton) number*

• in case these are conserved, the particle will decay by the *strong* interactions if:
  (1) all the decay products are *hadrons*
  (2) *flavour is conserved* (Q and B take care of u and d flavour, s only strangeness, c, and b need to be checked.)
conservation laws...

• if the above fails, but flavour is conserved and there are no $\nu$'s in the final state, then the decay can proceed by the \textit{EM interactions}.

• otherwise, the decay will normally proceed by the \textit{weak interactions}, (although if it does not go by simple W emission, it may be highly suppressed)

• interactions or scattering ($A+B \rightarrow C+D...$) follow the same rules except that there is no general kinematic constraint, since arbitrary energy can be put into the initial state.
weak interactions...

• recall $M_W = 80.4$ GeV
  $M_Z = 91.2$ GeV

• range of the weak force: $R_W \sim 2 \times 10^{-3}$ fm

• only couples with the left-handed fermions (right-handed anti-fermions)

• two basic W exchange processes:

$$\frac{g_W^2}{M_W^2} = \frac{4\pi\alpha_W}{M_W^2}$$

$$\alpha_W = 0.58\alpha$$

$$= 4.2 \times 10^{-3}$$
additional vertices

• the Z-boson vertices...

\[ \alpha_Z = \frac{1}{4\pi} \frac{g_Z^2}{M_Z^2} \]

\[ \frac{g_Z^2}{g_W^2} \frac{M_W^2}{M_Z^2} = \sin^2 \theta_W \]

\[ \sin^2 \theta_W = 0.223 \]

• the couplings of the W and Z bosons to fermions are tightly coupled.
interactions between bosons

- interactions between the W, Z bosons and photons.
weak propagator

- when a photon is exchanged, the process depends on the momentum transfer $1/q^2$

- the same principle stems for the weak interactions, except that the exchanged boson has a mass: $M_W$ or $M_Z$

\[ \text{Amplitude} \approx \frac{\sqrt{\alpha} X}{q^2 + M_X^2} \]
zero range approximation

- first introduced as a *point like* interaction with a zero range
- *Fermi Coupling constant* describes the probability of interaction:
  \[ G_F = 1.66 \times 10^{-5} \text{ GeV}^{-2} \]

\[
\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}
\]
example processes and cross sections

\[ \nu_e + e^- \rightarrow \nu_e + e^- \]

\[ \nu_e + e^+ \rightarrow \nu_e + e^+ \]

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]