Fermi’s Golden rule, nuclear decay in general, alpha- and cluster decay, UDL
See: Lecture Notes, Bertulani, Chapters 6 and 7 (B6-B7); S. Krane, Chapters 6 and 7 (K6&K8); B.A. Brown: Lecture Notes 2011 (BAB)
If you need data not given in the problem, refer the source where you found it (Please, learn using www.nndc.bnl.gov; you can even download a smartphone app, Nuclear Wallet Cards!).

1. Within the nuclear fission process, atomic nuclei far from the region of beta-stability are formed with independent formation probabilities $Q_1$ and $Q_2$ (for species 1 and 2, respectively). These nuclei decay with decay probabilities of $\lambda_{\beta}$- and $\lambda_\gamma$, respectively, to form stable nuclei.
   
   (a) Calculate $N_1$, $N_2^*$, and $N_2$ as functions of time (with initial conditions $N_1 = N_2^* = N_2 = 0$ at $t = 0$).
   
   (b) Is there a possibility that $N_2^*(t)$ decays with a pure $\exp(-\lambda_{\beta} t)$ decay law?

2. For some of the nuclei listed as ‘stable’ in the nuclide chart, the $\alpha$-decay is energetically possible, although a very rare process. An example of such a nuclide is $^{151}$Eu, and the half-life for its $\alpha$ decay to $^{147}$Pm ground state has been measured recently (2013). In this experiment, a total of 37.6±7.5 events, each releasing an energy of 1.949 MeV, were observed with an efficiency of 99.98%, during a measurement time of 462.2 h. The sample was a Li$_6$Eu(BO$_3$)$_3$ crystal (mass 6.16 g, of which 41.1% Eu), operated as a scintillation bolometer. Natural europium consists of two isotopes, $^{151}$Eu (47.81%) and $^{153}$Eu (52.19%).

   (a) Draw a level diagram illustrating the $\alpha$ decay of $^{151}$Eu to the states of $^{147}$Pm. Apply the selection rules for angular momentum and parity to determine the possible final states.
   
   (b) Estimate the $\alpha$-particle energy $E_\alpha$.
   
   (c) Calculate the half-life of $^{151}$Eu, assuming that $\alpha$-decay is the only decay mode.
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(d) Estimate the half-life with the Universal Decay Law (UDL) presented on Thursday’s lecture and compare with the result you got in (b).

3. All known 38 isotopes of Promethium (Pm) are unstable and it could be present on the Earth only as a temporal product created in U/Th fission, cosmogenic reactions induced by cosmic rays, radioactive decay, and neutron capture on other nuclides. Predominantly it is created in spontaneous fission (SF) of $^{238}\text{U}$, with yields of 1.88% ($^{147}\text{Ba}$), 2.32% ($^{147}\text{La}$) and 0.20% ($^{147}\text{Ce}$), via subsequent chain of radioactive decays $^{147}\text{Ba} \rightarrow ^{147}\text{La} \rightarrow ^{147}\text{Ce} \rightarrow ^{147}\text{Pr} \rightarrow ^{147}\text{Nd} \rightarrow ^{147}\text{Pm}$ (see the figure overleaf).

(a) Show that the overall yield of $^{147}\text{Pm}$ per SF of $^{238}\text{U}$, created in this way, is thus 4.4%.

(b) The $^{147}\text{Pm}$ activity in U ore was found as $(3 \pm 1) \times 10^{-4}$ Bq per 1 g of U. Show that this corresponds to $(4.5 \pm 1.5)$% yield per SF of $^{238}\text{U}$. The half-life of U is $4.468 \times 10^9$ years and the SF branching ratio is $5.4 \times 10^{-5}$.

(c) Taking into account the Uranium abundance of $2.7 \times 10^{-4}$% in the Earth crust and the mass of the crust $2.36 \times 10^{22}$ kg, calculate the equilibrium mass of natural Pm produced in SF.

(d) Assuming an abundance of 1.8 ppm of Eu in the Earth’s crust, would it be possible to estimate the equilibrium mass of Pm produced in the rare alpha decay in problem 2?

4. (Krane, Problem 12-11) The intensity of a source of thermal neutrons is to be measured by counting the induced radioactivity in a thin foil of indium metal exposed to the neutrons. The 1.0-$\mu$m thick foil has an area of $3.0 \times 3.0$ mm$^2$. The activation of $^{115}\text{In}$ to $^{116}\text{In}$ ($T_{1/2} = 54$ min) takes place with a cross-section of 160 b for thermal neutrons. After the foil is irradiated for 1.00 min, the counting cannot be started for 30 min. The efficiency of the detection system is only $2.4 \times 10^{-4}$ and in a 1 hour counting time $4.85 \times 10^4$ counts are accumulated.

(a) What is the flux of thermal neutrons on the foil (in units of neutrons/s per m$^2$)?

(b) Can you explain the 30 min “cooling time”, perhaps after consulting Appendix C of Krane?
Figure 1: For Problem 3