Interaction with matter

Understanding/description of interaction between matter and high energetic particles/radiation important:

- Principles of detection of particles/radiation
- Limitation of detectors
  - Efficiency
  - Position resolution
  - Energy resolution
  - Time resolution
- Impact on biological systems
  - Radiation damage
  - Radiation protection
  - Radiation therapy
- Literature
  - K. Kleinknecht, Teilchendetektoren
  - C. Grupen, Teilchendetektoren (BI Wissenschaftsverlag)
  - J. Ferbel, Experimental Techniques in High Energy Physics
  - Particle data group, chapter 26, http://pdg.lbl.gov/pdg.html
Interaction with matter

Energy loss per distance (dE/dx)

many different interactions, dominating processes depend on energy and particle type

Charged particles: defined reach
• Ionisation and excitation of electrons on shell → Bethe-Bloch formula
• Coulomb Scattering: Scattering in Coulomb field of nucleus small energy loss, but deflection
• Bremsstrahlung : dominant for low masses → radiation length

Photons: Absorption in matter, highly energy dependent – attenuation, no defined reach
• Photo electrical effect
• Compton scattering
• Pair production

Hadrons: inelastic scattering
• Hadron+nucleus → π±, K, p,n, fragments of nucleus
Energy loss of charged particles in matter

Energy loss due to ionisation and excitation

Energy loss per distance (dE/dx) : many theoretical works

- N. Bohr  Classical derivation
- Bethe, Bloch  Quantum mechanical description
- L. Landau  Charge distribution function
- E. Fermi  Density correction

...
Ionisation: classical derivation (Bohr)

Energy loss via inelastic collisions with electrons on shell → Ionisation and excitation

Assumptions:
- \(M(\text{particle}) \gg m_e\)
- Shell electron at rest (collision time small wrt orbital time)
- Projectile \(M, v = \beta c\), charge \(ze\), Target: charge \(Ze\)

Energy transfer onto target with mass \(m_t\): \(\Delta E = \Delta p/(2m_t)\)

Momentum transfer: time integral

\[
\Delta p = \int_{-\infty}^{+\infty} F_{\text{Coulomb}} \, dt
\]

\[
F_C = \frac{1}{4\pi \epsilon_0} \frac{zZe^2}{r^2}
\]

longitudinal forces cancel

\[
F_l(x) = -F_l(-x)
\]

only transversal forces important

\[
F_t = F_C \cdot \frac{b}{|\vec{r}|} = F_C \cdot \cos \theta
\]
Ionisation: classical derivation (Bohr) I

Transversal forces on projectile

\[ F_{Ct} = \frac{1}{4 \pi \epsilon_0} \frac{z Ze^2}{r^2} \cdot \cos \theta = \frac{1}{4 \pi \epsilon_0} \frac{z Ze^2}{b^2} \cdot \cos^3 \theta \]

**Momentum transfer** onto target

\[ \Delta p(b) = \int_{-\infty}^{+\infty} F_{Ct} \, dt = \int_{-\infty}^{+\infty} F_{Ct} \, \frac{dx}{v} = \frac{1}{4 \pi \epsilon_0} \frac{z Ze^2}{b^2} \int_{-\infty}^{+\infty} \cos^3 \theta \, \frac{dx}{v} \]

Integration →

\[ = \frac{1}{4 \pi \epsilon_0} \frac{z Ze^2}{b v} \int_{-\pi/2}^{+\pi/2} \cos \theta \, d\theta = \frac{1}{4 \pi \epsilon_0} \frac{2 z Ze^2}{b v} \]

**Energy transfer** onto target with mass \( m_t \)

\[ \Delta E(b) = \frac{\Delta p^2}{2 m_t} = \frac{2 z^2 e^4}{m_e c^2 \beta^2 b^2} \cdot \frac{1}{4 \pi \epsilon_0} \equiv 1 \]

shell electron \( Z = 1, m_t = m_e \)

1/m: collisions with nucleus may be neglected

Now: Sum over all shell electrons

Integration over all collision parameters \( b \)
Ionisation: classical derivation (Bohr) II

Energy loss due to collisions with electrons in tube (length $dx$, thickness $db$) at radius $b$:

$\text{# of electrons: } 2 \pi b \, db \, dx \, N_e \quad \text{electron density } N_e = N_A \frac{Z}{A} \rho$

$-dE(b) = \Delta E(b) N_e \, dV = \frac{4 \pi z^2 e^4}{m_e c^2 \beta^2} N_e \frac{db}{b} \, dx \quad (N_A \text{ Avogadro's constant, } A \text{ mass number})$

Integration over collision parameter from $b_{min}$ to $b_{max}$

$$\frac{-dE}{dx}(b) = \frac{4 \pi z^2 e^4}{m_e c^2 \beta^2} \rho \, N_A \frac{Z}{A} \frac{1}{b} \, db \quad \rightarrow \quad \frac{-dE}{dx} = \frac{4 \pi z^2 e^4}{m_e c^2 \beta^2} \rho \, N_A \frac{Z}{A} \ln \frac{b_{max} \beta}{b_{min}}$$

maximal energy transfer: central collision

$$\Delta E(b) = 2 \gamma^2 m_e \beta^2 c^2 = \frac{2 z^2 e^4}{m_e c^2 \beta^2 b^2}$$

minimal energy transfer: mean excitation energy $I$ (averaged excitation potential per electron in the target)

$$b_{min} = \frac{z e^2}{\gamma m_e c^2 \beta^2}$$

$$b_{max} = \frac{z e^2}{c \beta} \sqrt{\frac{2}{m_e I}}$$
(dE/dx) classical derivation (Bohr) III

\[-\frac{dE}{dx} = \frac{4\pi z^2 e^4 \rho}{m_e c^2 \beta^2} N_A \frac{Z}{A} \ln \frac{b_{\text{max}}}{b_{\text{min}}},\]

\[b_{\text{min}} = \frac{z e^2}{\gamma m_e c^2 \beta^2},\]

\[b_{\text{max}} = \frac{z e^2}{\beta c} \sqrt{\frac{2}{m_e I}}.\]

Classical formula for energy loss due to ionisation

\[-\frac{dE}{dx} = K \frac{z^2 \rho}{\beta^2} \frac{Z}{A} \frac{1}{2} \ln \left( \frac{2 c^2 \gamma^2 \beta^2 m_e}{I} \right),\]

\[K = \frac{4\pi e^4}{c^2 m_e}, \quad N_A = 0.31 \text{ MeV cm}^2/\text{g}\]

Unit \([dE/dx] = \text{MeV /cm}\]

Warning: usually energy loss is divided by density: \([dE/dX] = \text{MeV cm}^2/\text{g}\]

\[\rightarrow dE/dX \text{ almost independent of material }\]
\[\text{only material dependence through } Z/A \text{ and } \ln(1/I)\]
(dE/dx) Bethe Bloch Formula

full quantum mechanical derivation: Bethe-Bloch Formula

\[-\frac{dE}{dX} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \left( \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right] \]

\[K = \frac{4 \pi e^4}{c^2 m_e} N_A = 0.31 \text{ MeV cm}^2/\text{g} \]

I mean excitation energy
Z atomic number
A mass number of absorber
Validity: 6 MeV < E < 6 GeV (π), generally 0.02 < β < 0.99 (1% accuracy!!)

\[T_{max}: \text{max kinematic energy transferred} \]
\[z: \text{Charge of particle} \quad dE/dX \propto z^2 \]

Corrections

• δ(β) Density correction due to polarisation, important for high energies
• C/Z Correction close to shell boundaries, relevant for small energies

1% @ βγ = 0.3
(dE/dx) Bethe-Bloch-(Sternheimer) Formula

classical result: \[
\frac{-dE}{dx} = \frac{K z^2 Z}{\beta^2} \frac{1}{A} \frac{2 m_e c^2 \gamma^2 \beta^2}{I} \ln \frac{2 m_e c^2 \gamma^2 T_{max}}{I^2}
\]

full quantum mechanical derivation: Bethe-Bloch Formula

\[
\frac{-dE}{dx} = K z^2 Z \frac{1}{A} \left[ \frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \delta - \frac{C}{Z} \right]
\]
describes mean energy loss or stopping power

- \( T_{max} \): max kinematic energy transferred from particle (M) onto electron

\[
T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2 \gamma m_e / M + (m_e / M)^2} \approx 2 m_e c^2 \beta^2 \gamma^2 \quad \gamma m_e \ll M
\]

- small energies and particles with high masses: result equals classical result except factor 2 (imperfect description of very far collisions: binding of electrons cannot be neglected)

- independent of mass of particle ! (\( \gamma m_e \ll M \))
- Unit MeV/(g/cm²) \( dx = \rho \, ds \) is material occupancy
- \( dE/dX \) almost independent of target material
- Formula needs to be modified for electrons
- Bethe-Bloch formula is not valid for slow particles \( \beta \gamma < 0.02 \) where \( dE/dX \propto \beta \)
Energy loss $dE/dX$

$dE/dx$ of muons in copper

- $\beta \gamma < 3 \Rightarrow dE/dX \propto 1/\beta^2$
- $\beta \gamma \approx 3.5$ minimum $dE/dX \approx 1-1.7$ MeV cm$^2$/g
- $\beta \gamma > 3.5$ logarithmic rise $dE/dX \approx 2$ MeV cm$^2$/g
- $\beta \gamma > 1000$ Bremsstrahlung dominant
- very low energy: BB not valid, empirical models
- low energies: shell corrections
- high energies: density corr.

Minimum: $\beta \gamma$ 3-4 minimum ionising particle (MIP) looses $\sim 1-1.7$ MeV/(g/cm$^2$)

PDG  http://pdg.lbl.gov/pdg.html
Shell correction

\[-\frac{dE}{dX} = K \frac{Z^2}{A} \frac{Z}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} - \beta^2 - \frac{\delta}{2} \frac{C}{Z} \right]\]

Shell corrections \((C/Z)\) constitute a correction to slow particles (e.g. for protons in the energy range of 1-100 MeV)

Correction for the assumption that particle velocity >> bound electron velocity

Assumption that electron is at rest not valid

Maximal about 6%

Calculation with various approximations


http://www.srim.org/SRIM/SRIMPICS/IONIZ.htm

Copper: about 1% at \(\beta \gamma = 0.3\) (6 MeV Pion), decreases rapidly with velocity
Mean excitation energy $I$

\[-\frac{dE}{dX} = K \frac{z^2 Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \left( \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} \right) - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right] \]

$I$ characteristic for target material \[ I = 16 Z^{0.9} \text{ eV} \text{ for } Z > 1 \]

Mean excitation energy > ionisation E
Theoretical calculation of $I$ has a long history.
Summaries can be found in several reviews:

Example Argon: $Z = 18$, $I = 215$ eV, measured 190.8 eV (Ionisation energy 15.7 eV)
Charge dependence

\[
\frac{-dE}{dX} = K Z^2 Z \frac{1}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} - \frac{\beta^2}{2} - \frac{\delta}{2} - \frac{C}{Z} \right]
\]

Tracks of ions in emulsion

Energy loss depends quadratically on charge of projectile
width allows estimate of charge
\[ \frac{-dE}{dX} = K \frac{Z^2 Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right] \]

- depends on velocity not on mass → particle identification
- \( \beta \gamma \) small \( dE/dx \propto 1/\beta^2 \)
- \( \beta \gamma \approx 3.5 \) broad minimum
  - light absorbers: \( Z/A \approx 0.5 \)
- \( dE/dx(\text{min}) \approx 1.5 \text{ MeV}/(\text{g/cm}^2) \)
  → minimal ionising particles (MIP)
- \( \beta \gamma > 4 \) \( dE/dx \propto 2 \ln(\beta \gamma^2) \)
  - logarithmic (relativistic) rise

\[ \beta \gamma \approx 3-4 \]
$dE/dx_{\text{min}}$ different materials

\[ \frac{-dE}{dX} = K \frac{Z^2}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} \right] - \frac{\beta^2}{2} - \frac{\delta}{Z} - C \]

$dE/dX$ depends on $A$, $Z$ of target material

$\beta \gamma \approx 3.5$ broad minimum

$\rightarrow$ minimum ionising particles (MIP)

$H_2$ $Z/A \approx 1$ $dE/dX_{\text{min}} \approx 4 \text{ MeV}/(\text{g/cm}^2)$

others $Z/A \approx 0.5$ $dE/dX_{\text{min}} \approx 2 \text{ MeV}/(\text{g/cm}^2)$

$\frac{dE}{dX_{\text{min}}} \approx 1-1.7 \text{ MeV}/(\text{g/cm}^2)$

only weak material dependence

PDG http://pdg.lbl.gov/pdg.html
$dE/dX$ at minimum

$\beta\gamma \rightarrow 3.5$ broad minimum \quad $dE/dX_{\text{min}} \approx 1\text{-}1.7 \text{ MeV}/(\text{g/cm}^2)$

only small material dependence

\[ -dE/dx_{\text{min}} \text{ (MeV g}^{-1}\text{cm}^2) \]

\[ Z \quad \text{H} \quad \text{He} \quad \text{Li} \quad \text{Be} \quad \text{B} \quad \text{CNO} \quad \text{Ne} \quad \text{Fe} \quad \text{Sn} \]

\[ 1 \quad 2 \quad 5 \quad 10 \quad 20 \quad 50 \quad 100 \]

$dE/dX(\text{min})$ for different chemical elements fitted by a straight line $Z>6$
Relativistic Rise

Lorentz-contraction of field lines
transversal component of E-field grows with \( \gamma \)
\( \rightarrow \) larger collision distances, more collisions

Saturation for high energies \( \rightarrow \) Fermi plateau \( T_{\text{max}} = E \)
Solids \( (dE/dX)_{\beta \rightarrow 1} \approx 1.05-1.1 \) \((dE/dX)_{\text{MIP}}\)
Gases \( (dE/dX)_{\beta \rightarrow 1} \approx 1.5 \) \((dE/dX)_{\text{MIP}}\)

\( \delta \)-correction, density effect:
E-field gets partially shielded for high densities due to polarisation
\( \delta(\gamma) = 2 \ln \gamma + \zeta \) (\( \zeta \) material constant)
\( \rightarrow \) less collisions with far distant electrons, smaller \( dE/dX \)

\( T_{\text{max}} = E \)
Particle identification

\[-dE/dx \approx K z^2 Z \frac{1}{A} \frac{2m_e c^2 \beta^2 \gamma^2}{I} \left[ \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2}{I} \right) - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right] \text{ as long as } 2 \gamma m_e \ll M\]

\[
dE/dx \text{ only depends on velocity not on mass of particle } \rightarrow \text{ Particle identification}
\]

ALICE TPC measured energy loss

- Heavy particles
  \(dE/dx\) well described by Bethe-Bloch formula
- Electrons not described by Bethe-Bloch

\[
s - p - K - \pi - \mu - e - d
\]

http://aliceinfo.cern.ch/
ALICE TPC

88 m³ cylinder filled with gas, 5.1 m long

Divided in two drift regions by the central electrode located at its axial centre.

Uniform electric field along the z-axis electrons drift towards the end plates

Signal amplification: avalanche effect near anode wires

Readout: 570132 pads in cathode of multi-wire proportional chamber

http://aliceinfo.cern.ch/Public/en/Chapter2/Chap2_TPC.html

Cosmic muon in TPC
Restricted Bethe Bloch

Particle detectors don't measure energy loss, but energy deposited in detector.

Eg. highly energetic electrons may leave the detector → measured energy is too small

Restricted Bethe Bloch: Cut off parameter $T_{cut}$: dE with $T > T_{cut}$ are neglected

$$
-dE = K Z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2 m e c^2 \beta^2 \gamma^2 T_{upper}}{I^2} - \beta^2 \left( 1 + \frac{T_{upper}}{T_{cut}} \right) - \frac{\delta}{2} - \frac{C}{Z} \right]
$$

$$
T_{upper} = \min(T_{cut}, T_{max})
$$
Range in matter

Formally: integration of Bethe-Bloch formula

\[ R = \int_0^E \frac{dX}{dE} \, dE \]

statistical process → mean penetration depth

range in cm: divided by density!

Example 1 GeV particle in lead

K⁺  \( M = 493.6 \text{ MeV} \quad \beta\gamma = 2.02 \)
\[ R/M = 800 \text{ g cm}^{-2} \text{ GeV}^{-1} \]
\[ R = 395 \text{ g cm}^{-2} \]
\[ ds = R = 35 \text{ cm in lead} \]

Myon  \( R/M = 7000 \text{ g cm}^{-2} \text{ GeV}^{-1} \)
\[ ds = 64 \text{ cm} \]

Proton  \( R/m = 220 \text{ g cm}^{-2} \text{ GeV}^{-1} \)
\[ ds = 18 \text{ cm} \]
Ionisation tracks

Energy loss largest for low energy particles $\rightarrow$ increase of energy deposits (ionisation) towards end of range (Bragg peak)

$\delta$-(knock-on) electrons: have high enough energy for ionisation track

Tracks of mono energetic $\alpha$ particles in cloud chamber: range 8.6 cm, small variation
Range: Bragg curve

Bragg curve: describes energy loss of particle as function of penetration depth

Particle in matter → particle de-accelerates → energy loss increases as particle gets slower

Bragg-Peak: largest energy deposit at end of the track important for radiation therapy!

70 MeV protons in water

Penetration depth
Possible to precisely deposit dose at well defined depth by varying beam energy

Application in medicine: Concentration of energy deposit at end of reach allows treatment of tumours with moderate exposure dose for the surrounding tissue, contrary to x-ray

Ionisation profile for $^{12}$C ions in water
Proton therapy at PSI

Spread out Bragg peaks (SOBP)
Different absorbers → almost constant dose in region of tumour
Energy loss (Straggling)

Energy loss is statistical process \[ \Delta E = \sum_{i=0}^{N} \delta E_i \]  
N: number of collisions

- ionisation loss distributed statistically
- collisions with small \(dE\) more probable
- large \(dE\) rare \(\rightarrow \) electrons with keV (\(\delta\)-electrons)
- \(\delta\)-rays have enough energy for ionisation
- asymmetry: mean energy loss > most probable energy loss

Parametrisation: asymmetric Landau distribution

Landau-Tail: rare interactions with large energy transfer
Tails up to the kinematic maximum
Bethe-Bloch gives mean energy loss

Thick layers or dense material: Gaussian distr.

\(dEdX\): most probable value \(\neq\) mean energy loss
Landau distribution

\[ \Delta E_{mp} = 82 \text{ keV} \]

\[ \Delta E_{mp} = 56.5 \text{ keV} \]

Landau: Probability \((\Delta E) = \frac{1}{2\pi \xi} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda})\right) \]

\[ \lambda = (\Delta E - \Delta E_{mp})/\xi \]

\(\xi\): material constant mean energy loss in layer \(x\)

\(\xi = K \rho x / \beta^2\) depends on density, thickness and velocity

\(\Delta E\) mean E-loss, \(\Delta E_{mp}\) most probable E-loss

“Theory” 300 \(\mu\text{m} \text{ Si}\)

\(<\Delta E> \sim 115 \text{ keV}\)

\(\xi = 26 \text{ keV}\)

Measurement

Includes a Gaussian electronics noise contribution of 2.3 keV
δ Electrons

Energy distribution of secondary electrons:

\[ F(T) = 1 - \beta^2 \frac{T}{T_{\text{max}}} \quad \text{(Spin 0)} \]

\[
\frac{d N^2}{dT dx} = \frac{1}{2} k \frac{z^2 Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2}
\]

Example: 500 MeV pion in 300 μm Si: 5% produce an electron with \( T > 166 \) keV
important background in Cherenkov counter
Number of δ-electrons proportional \( z^2/\beta^2 \)
**δ Electrons**

1969 Mc Cusker: Evidence of quarks in air shower cores (Phys Rev Lett)
Narrow tracks in cloud chambers: low ionisation $\rightarrow$ charge $< 1!$

Number of δ electrons proportional to $z^2/\beta^2$
Summary $dE/dx$ through ionisation

described by Bethe-Bloch Formula

- only small material dependence
- Energy loss $\propto z^2$ (particle)
- independent of mass $\rightarrow$ particle identification
- most of the energy is deposited towards the end of the range

- $\beta\gamma < 3 \ dE/dX \propto 1/\beta^2$
- $\beta\gamma \approx 3.5$ minimum $dE/dX$ 1-1.7 MeV cm$^2$/g
- $\beta\gamma > 3.5$ logarithmic rise $dE/dX \approx 2$ MeV cm$^2$/g
- $\beta\gamma > 1000$ Bremsstrahlung dominant
- Bethe-Bloch is not valid for slow particles ($\beta\gamma<0.05$) ($dE/dX \propto \beta$) here only phenomenological models available
Addtional Material: Ranges

Range of electrons, protons and $\alpha$ particles in air and water

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<th>Teilchen</th>
<th>Energie [MeV]</th>
<th>Reichweite Luft [m]</th>
<th>Reichweite Wasser [m]</th>
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Additional Material: Ionisation yield

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<th>Material</th>
<th>Density g/cm³</th>
<th>$E_{\text{ion}}$ [eV]</th>
<th>$I$ [eV]</th>
<th>$W$ [eV]</th>
<th>$n_p$ [cm⁻³]</th>
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<td></td>
<td>3.6</td>
<td>172</td>
<td></td>
<td>1000000</td>
</tr>
</tbody>
</table>

- $W = E/n_t$ mean energy per electron-ion pair
- $n_p$: Primary ionisation: directly produced electron-ion pairs $\sim 1.45 Z$
- $n_t = n_p$ +secondary ionisation: add. electron-ion pairs through primary electrons
- $W$: mean energy for production of electron-ion pair
- $W$ higher than ionisation energy

- Semiconductors, $W$ small, many electron-ion pairs, good resolution, small detectors
Since BB applies to pure elements, direct measurements are needed in compounds for accurate values.

Good approximation: weighted sum of loss rates of constituents weighted according to fraction $a_i$ of electrons

\[
\frac{1}{\rho} \frac{dE}{dx} = \sum \frac{w_i}{\rho_i} \left( \frac{dE}{dx} \right)_i \quad w_i = \frac{a_i A_i}{A_{eff}}
\]

\[
Z_{eff} = \sum a_i Z_i \\
A_{eff} = \sum a_i A_i
\]

$a_i$ is the molar fraction of element $i$ with atomic weight $A_i$.

$A_{eff}$ is the atomic weight of all constituents

Tables for density and shell correction