7 Dark matter

7.1 Observational evidence for dark matter

The term *dark matter* was coined by Jacobus Kapteyn in 1922 in his studies of the motions of stars in our galaxy to refer to matter that interacts gravitationally, but is not seen via electromagnetic radiation [1]. He found that no dark matter is needed in in the galactic Solar neighbourhood. In 1932, Jan Oort made the contrary claim that there is twice as much dark matter as visible matter in the Solar vicinity. This is the first claimed evidence for dark matter. However, later observations have shown it to be wrong, and the discovery of dark matter is usually credited to Fritz Zwicky who made the first correct argument for the existence of dark matter in 1933. Zwicky concluded from measurements of the redshifts of galaxies in the Coma cluster that their velocities are much larger than the escape velocity due to the visible mass of the cluster.

There are nowadays large amounts of evidence for dark matter, including from gravitational lensing, expansion rate of the universe and other observations. One of the earliest, and easiest to understand, pieces of evidence comes from *rotation curves* of galaxies, which have been studied extensively since the 1970s, notably by Vera Rubin. According to Newtonian gravity, the velocity v of a body on a circular orbit in an axially symmetric mass distribution is

$$\frac{v^2}{r} = G_{\rm N}\frac{M(r)}{r^2} \ , \tag{7.1}$$

where $M(r)$ is the mass inside radius r, and the function $v(r)$ is called the *rotation curve*. For an orbit around a compact central mass, for example planets in the Solar system, we get $v \propto r^{-1/2}$, in agreement with Kepler's third law. For stars orbiting the centre of a galaxy the situation is different, since the mass inside the orbit increases with the distance. Suppose that the energy density of a galaxy decreases as a power-law,

$$\rho \propto r^{-n} \tag{7.2}$$

with some constant n. Then the mass inside radius r is

$$M(r) \propto \int \mathrm{d}r r^2 r^{-n} \propto r^{3-n} \qquad \text{for} \quad n < 3 \ . \tag{7.3}$$

Thus the rotation velocity in our model galaxy should vary with distance from the centre as

$$v(r) \propto r^{1-n/2} \ . \tag{7.4}$$

Observed rotation curves increase with r for small r, i.e., near the centre of the galaxy, but then typically flatten out, so that $v(r) \approx$ const.. According to (7.4), this would indicate the density profile

$$\rho \propto r^{-2} \ . \tag{7.5}$$

However, the density of stars falls more rapidly away from the core of a galaxy, and goes down exponentially at the edge. Also, the total mass from stars and other visible objects, like gas and dust clouds, is too small to account for the rotation velocity at large distances.

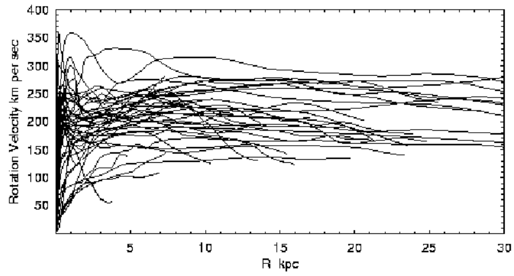

Figure 1: Rotation curves of spiral galaxies, from [2].

This seems to indicate the presence of another mass component to galaxies. This mass component should have a different density profile than the visible, or *luminous*, matter, so that it would be subdominant in the inner parts of the galaxy, but would dominate in the outer parts. The dark component appears to extend well beyond the visible parts of galaxies, forming a dark *halo* surrounding the galaxy.

More detailed observations indicate that instead of of $1/r^2$, the distribution of dark matter in galaxies is well fit by the *Navarro-Frenk-White (NFW) profile*,

$$\rho = \frac{\rho_0}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2} \ , \tag{7.6}$$

where ρ_0 and r_s are constants. The profile obviously does not hold all the way to the centre (the physical density is finite everywhere). Near the centres of galaxies, the densities are typically dominated by baryonic matter, and the dark matter profile rises less steeply than in the NFW case.

Dark matter can be discussed in terms of the *mass-to-light ratio* M/L of sources. It is customarily given in units of $M_\odot/L_\odot$, where $M_\odot$ and $L_\odot$ are the mass and absolute luminosity of the Sun. The luminosity of a star increases with its mass faster than linearly, so stars with $M > M_\odot$ have $M/L < 1$, and smaller stars have $M/L > 1$. Small stars are more common than large stars, so a typical mass-to-light ratio from the stellar component of galaxies is $M/L \sim$ a few. For stars in our part of the Milky Way galaxy, $M/L \approx 2.2$. Because large stars are more short-lived, M/L decreases with the age of the star system, and the typical M/L from stars in the universe is somewhat larger. However, this still does not account for the full masses of galaxies.

The mass-to-light ratio of a galaxy turns out to be difficult to determine; the larger volume around the galaxy you include, the larger M/L you get. The mass M is determined from velocities of orbiting bodies and at large distances there may be no such bodies visible. For galaxy clusters you can use the velocities of the galaxies themselves as they orbit the centre of the cluster. The mass-to-light ratios of clusters appear to be several hundreds. Presumably isolated galaxies would have similar values if we could measure them to large enough radii.

Estimates for the total matter density Ω_{m0} based on the gravitational effects of matter in the universe via many different methods give a similar conservative range $0.1 \lesssim \Omega_{m0} \lesssim 0.4$, with a more likely range of $0.15 \lesssim \Omega_{m0} \lesssim 0.3$ [3]. Also, from the CMB (combined with other data) we have $\omega_m = \Omega_{m0}h^2 = 0.1413 \pm 0.0011$ for the spatially flat ΛCDM model [4], and $\omega_m = 0.14 \pm 0.01$ model-independently [5][1].

The estimates for the amount of ordinary matter in the objects we can see on the sky, stars and visible gas and dust clouds, i.e. luminous matter, give a much smaller contribution to the density parameter,

$$\Omega_{\mathrm{lum0}} \lesssim 0.01 \tag{7.7}$$

In the previous chapter we found that big bang nucleosynthesis leads to the value $0.021 \leq \Omega_{b0}h^2 \leq 0.025$ at 95% C.L., and the CMB gives a similar range. For $h = 0.7$, we get

$$\Omega_{b0} = 0.04 \ldots 0.05 \ . \tag{7.8}$$

We thus have, at very high confidence,

$$\Omega_{\mathrm{lum0}} < \Omega_{b0} < \Omega_{m0} \ . \tag{7.9}$$

This is consistent, as all luminous matter is baryonic, and all baryonic matter is matter. That we have two inequalities tells us that there are two kinds of *dark matter* (as opposed to luminous matter): baryonic dark matter (BDM) and nonbaryonic dark matter. We do not know the precise nature of (all) non-baryonic dark matter, and this is called the *dark matter problem*. Determining the nature of dark matter is one of the most important problems in cosmology today. Often the expression "dark matter" is used to refer to the nonbaryonic kind only, or only to non-baryonic dark matter other than neutrinos, i.e. only to the part whose nature remains unknown.

7.2 Baryonic dark matter

Candidates for BDM include compact (i.e. planet-like) objects in interstellar space as well as thin intergalactic gas (or plasma). Objects of the former kind have been dubbed MACHOs (Massive Astrophysical Compact Halo Objects) to contrast them with another dark matter candidate, WIMPs, to be discussed later. A way to detect such a dark compact object is *gravitational microlensing*: if such a massive object passes near the line of sight between us and a distant star, its gravity focuses the light of that star towards us, and the star appears to brighten for a while. The brightening has a characteristic time profile, and is independent of wavelength, which clearly distinguishes it from other ways a star may brighten (variable stars).

An observation of a microlensing event gives an estimate of the mass, distance and velocity[2] of the compact object, but tells nothing else about it. Thus in principle we could have nonbaryonic MACHOs. But as we do not know of any such objects (except black holes), the MACHOs are usually thought of as ordinary substellar objects, such as *brown dwarfs* or "jupiters". Ordinary stars can of course also cause a microlensing event, but then we would also see light from the star. Heavier relatively faint objects that could fall into this category include old white dwarfs, neutron stars and black holes.

[1] In fact, if there is only baryonic matter, the CMB anisotropy pattern looks qualitatively different than in the case with dark matter, so the CMB provides a strong case for dark matter even in the absence of any other observations. We return to this in the second part of the course.

[2] Actually we do not get an independent measure of all three quantities, as the observables depend on combinations of these. However, we can make some reasonable assumptions of the expected distance and velocity distributions among such objects, leading to a rough estimate of the mass. Especially from a set of many events, we can get an estimate for the typical mass.

The masses of ordinary black holes are included in the Ω_b estimate from BBN, since they were formed from baryonic matter after BBN. However, if there are *primordial black holes* produced before BBN, they would not be included in Ω_b.

A star requires a mass of about $0.07M_\odot$ to ignite thermonuclear fusion, and to start to shine. Smaller, "failed", stars are called *brown dwarfs*. They are not completely dark; they are warm balls of gas that radiate faint thermal radiation. They were warmed up by the gravitational energy released in their compression to a compact object. Thus brown dwarfs can be, and have been, observed with telescopes if they are close. Smaller such objects are called "jupiters" after their representative in the Solar System.

The strategy to observe a microlensing event is to monitor constantly a large number of stars to catch such a brightening when it occurs for one of them. Since the typical time scales of these events are many days, or even months, it is enough to look at each star, say, once every day or so. As most of the dark matter is in the outer parts of the galaxy, further out than we are, it would be best if the stars to be monitored were outside of our galaxy. The Large Magellanic Cloud, a satellite galaxy of our own galaxy, is a good place to look for, being at a suitable distance, where individual stars are still easy to distinguish. Because of the required precise alignment of us, the MACHO, and the distant star, the microlensing events will be rare. But if the BDM in our galaxy consisted mainly of MACHOs (with masses between that of Jupiter and several solar masses), and we monitored constantly millions of stars in the LMC, we should observe many events every year.

Such observing campaigns (MACHO, OGLE, EROS, ...) were begun in the 1990's. Indeed, several microlensing events have been observed. The typical mass of these MACHOs turned out to by $\sim 0.5M_\odot$, much larger than the brown dwarf mass that had been expected. The most natural faint object with such a mass would be a white dwarf. However, white dwarfs had been expected to be much too rare to explain the number of observed events. On the other hand the number of observed events is too small for these objects to dominate the mass of the BDM in the halo of our galaxy.

BDM in our universe is dominated by thin intergalactic ionised gas. In fact, in large clusters of galaxies, we can see this gas, as it has been heated by the deep gravitational well of the cluster, and radiates X-rays.

7.3 Nonbaryonic dark matter

The favourite candidates for nonbaryonic dark matter can be divided into three classes, hot dark matter (HDM), warm dark matter (WDM) and cold dark matter (CDM), based on the typical velocities of the particles making up this matter at the time they decouple from the thermal bath.

Dark matter particles are called HDM if they decouple while they are relativistic and the number density is determined by the freeze-out of their interactions (we will discuss this shortly)[3]. Then they retain a large number density, requiring their masses to be small, less than 100 eV, so that the dark matter density would not be too high. Because of the small mass, their thermal velocities are large when structure formation begins, making it difficult to trap them in potential wells of the forming structures. CDM, on the other hand, refers to dark matter particles with

[3] Today, the HDM particles should be nonrelativistic in order to count as "matter".

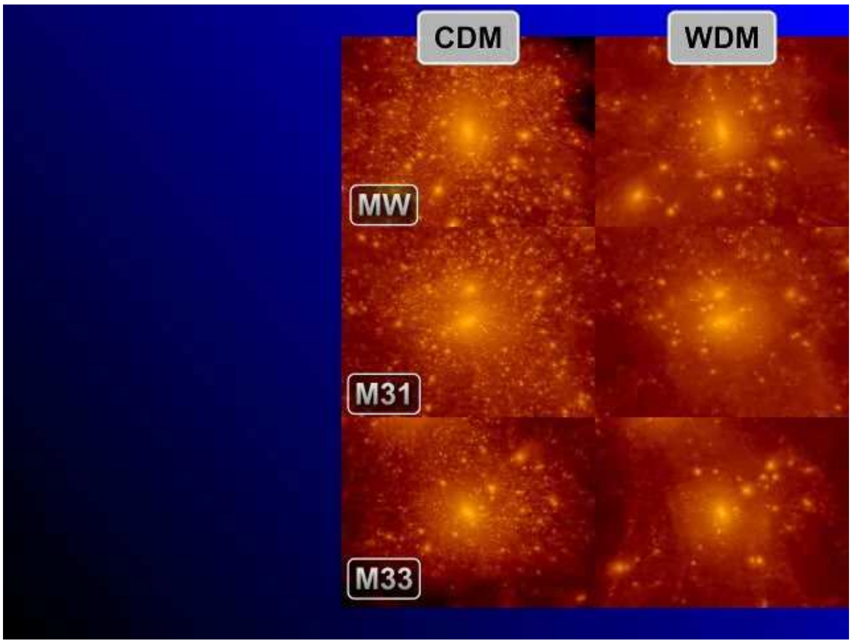

Figure 2: Comparison of the expected halo of the Milky Way and the galaxies M31 and M33 in CDM and WDM models. From http://www.clues-project.org/images/darkmatter.html.

negligible velocities. If these velocities are thermal, this requires their masses to be large, which means that they must have decoupled while already nonrelativistic, so their number density would have been suppressed by annihilation. Candidates between hot and cold are called, naturally enough, warm dark matter.

HDM, WDM and CDM all have a different effect on structure formation in the universe. Structure formation refers to the process in which the originally nearly homogeneously distributed matter forms bound structures such galaxies and galaxy clusters under the pull of gravity. We shall discuss structure formation in the second part of the course. But let us already mention that today the best way to differentiate between HDM, WDM and CDM is through the observed *large-scale structure* in the universe, i.e., the way galaxies are distributed in space, combined with the CMB which shows the seeds of structure. We show in figure 2 the results of a simulation of the halo of dark matter around the Milky Way and two other galaxies. For CDM, there is more substructure and satellites around the galaxy, while their formation is suppressed for WDM. According to observations, there is an order of magnitude less satellites observed around the Milky Way than predicted by CDM models. However, the observations are not complete, and the discrepancy may also be due to other causes than WDM.

The most common candidates for non-baryonic dark matter are Weakly Interacting Massive Particles, or WIMPs. They decouple from the thermal bath of the early universe early, like neutrinos, but are much heavier, so that they are a form of CDM. The interactions of some dark matter candidates are stronger or weaker than those of WIMPS. For example, *gravitinos* have only gravitational-strength interactions, while TIMPs (Technicolour Interacting Massive Particles) can interact strongly.

7.4 Hot dark matter

The archetypal HDM candidates are neutrinos, which have a small but nonzero rest mass. The cosmic neutrino background would make a significant contribution to the total density parameter today if the neutrinos had a rest mass of the order of 1 eV or above.

For massive neutrinos, the number density today is the same as for massless neutrinos, but their energy density today is dominated by their rest masses, giving (there is a factor of 3/4 since neutrinos are fermions and and 4/11 due to e^+e^--annihilation)

$$\rho_\nu = \sum_i m_{\nu_i} n_{\nu_i} = \frac{3}{11} n_\gamma \sum_i m_{\nu_i} \ , \tag{7.10}$$

where the sum is over the neutrino mass eigenstates (which are not the same as the weak interaction eigenstates, for whom the names electron neutrino, muon neutrino and tau neutrino are properly reserved). For $T_0 = 2.725$ K, this gives the neutrino density parameter

$$\Omega_{\nu 0} h^2 = \frac{\sum_i m_{\nu_i}}{94.14 \text{ eV}} \ , \tag{7.11}$$

which applies if the neutrino masses are less than the neutrino decoupling temperature, 1 MeV, but greater than the present temperature of massless neutrinos, $T_{\nu 0} = 0.168$ meV. This counts then as one contribution to Ω_{m0}. In this case, neutrinos are *hot dark matter* (HDM). Data on large scale structure combined with structure formation theory requires that a majority of the matter in the universe has to be CDM. A conservative upper limit at the moment from CMB observations is [4]

$$\sum_i m_{\nu_i} \lesssim 0.7 \text{ eV} \ . \tag{7.12}$$

Therefore the maximum contribution of neutrinos is $\Omega_{\nu 0} h^2 \lesssim 0.007$, about one order of magnitude below baryonic matter.

If neutrinos were the dominant form of matter, there would be a lower limit on their mass from constraints on the phase space density, called the *Tremaine–Gunn limit.* Essentially, in order to achieve a certain rotation velocity for galaxies, you need a certain amount of mass inside a given volume, and the Pauli exclusion principle constrains the number number of particles you can pack inside a given volume. Even though we know that neutrinos are a subdominant component of dark matter, the Tremaine-Gunn limit applies to any fermionic dark matter candidate, even if its distribution is not thermal. There is no such lower limit on the mass of a bosonic dark matter particle.

Exercise. Suppose neutrinos would dominate the mass of galaxies (to the extent you could ignore all other forms of matter). We know the mass of a galaxy (within a certain radius) from its rotation velocity. The mass could come from a smaller number of heavier neutrinos or a larger number of lighter neutrinos, but the available phase space (you don't have to assume a thermal distribution) limits the total number of neutrinos, whose velocity is below the escape velocity. This leads to a lower limit of the neutrino mass m_ν. Assume for simplicity that either **a**) all neutrinos have the same mass, or **b**) only ν_τ is massive. Let r be the radius of the galaxy, and v its rotation velocity at this distance. Find the minimum m_ν needed for neutrinos to dominate the galaxy mass. (A rough estimate is enough: you can, e.g., assume that the neutrino distribution is spherically symmetric, and that the escape velocity within radius r equals the escape velocity at r.) Give the numerical value for the case $v = 200$ km/s and $r = 10$ kpc.

7.5 Cold dark matter

Observations of large-scale structure together with the theory of structure formation requires that dark matter is dominated by CDM or WDM, with CDM being the currently preferred option. There is no particle in the Standard Model of particle physics that is suitable as CDM. We can therefore say that cosmological observations of dark matter are one of the most important pieces of evidence for physics beyond the Standard Model.

A major class of CDM particle candidates is called WIMPs (Weakly Interacting Massive Particles). For a HDM candidate, the mass must be small so that the total contribution to the energy density today would not be huge; from (7.11) we see that a neutrino mass larger than about a dozen eV would give more energy density than observed. In contrast, if the mass of a weakly interacting particle species is much larger than the decoupling temperature of weak interactions, these particles are largely annihilated before the decoupling. This suppression of the number density makes it possible to achieve a suitable energy density starting from a thermal distribution at very high temperatures.

A favourite WIMP candidate comes from supersymmetric extensions of the Standard Model. In the simplest version, the Minimal Supersymmetric Standard Model (MSSM), every Standard Model particle has a partner with the same quantum numbers[4] but a spin which is different by 1/2. The MSSM has a symmetry called R-parity as a result of which superpartners can only be created or destroyed in pairs, so the lightest supersymmetric partner (LSP) is stable. The parameters of the MSSM can be chosen such that the LSP is electrically neutral and a col or singlet, so that it has only weak interactions. If it exists, it is possible that the LSP would be created and detected at the LHC (Large Hadron Collider) at CERN. A measurement of its properties would allow a calculation of its expected number and energy density in the universe. Thus far, there has been no evidence for (or even suggestions of) MSSM, or any other physics beyond the Standard Model, at the LHC.

If a CDM particle was in thermal equilibrium in the early universe, its number density is suppressed, as noted above. Its mass then has to be large to have a significant energy density today. (We will soon look at this in detail!) In the MSSM, the LSP is expected to have a mass somewhere in the range between 100 GeV to a few TeV or so. However, if the particle was not in thermal equilibrium when it decoupled, the number density is not thus constrained.

For the particle not to be in thermal equilibrium, its interactions need to be very weak, and typically it should not even feel the weak interaction (which, despite the name, is not actually weak at large energies; recall that the weak interaction cross section is $\propto E^2$). One such candidate is called the *axion*. Axion particles are born with small velocities and have never been in thermal equilibrium. They are related

[4]If supersymmetry were unbroken, the mass would also be the same. In that case superpartners would have been observed already, so supersymmetry has to be broken. The partners retain the same quantum numbers, but their masses become different.

to the so-called “strong CP problem” in particle physics. We will not go into the details of this, but it can be phrased as the question “why is the neutron electric dipole moment so small?”. (The electric dipole moment is zero to the accuracy of measurement, the upper limit being $d_n < 0.29 \times 10^{-25} e\text{cm}$ [6], whereas the neutron does have a significant magnetic dipole moment.) A proposed solution involves an additional symmetry of particle physics, the Peccei–Quinn symmetry. The axion would is the Goldstone boson of the breaking of this symmetry. The important point for us is that these axions would be created in the early universe when the temperature falls below the QCD energy scale (of the order of 100 MeV), with negligible kinetic energy. Thus axions would have negligible velocities, and act like CDM. (Though calling axions “cold” is bit of a misnomer, as their phase space distribution is not thermal! Here the word just means that the typical kinetic energy is much smaller than the mass.) Another dark matter candidate of this type is the *gravitino*, the supersymmetric partner of the graviton. We will not discuss this type of dark matter further, and will stick to massive CDM particles.

7.6 Dark matter decoupling

Many dark matter candidates, WIMPs in particular, are in thermal equilibrium at early times and decouple once their interactions become too weak to keep them in equilibrium. Such particles are called *thermal relics*, since their density today is determined by the thermal equilibrium of the early universe, just as is the case for neutrinos. If the candidate is stable (or has a long lifetime) and there are no particles decaying to it, the number of particles is conserved after decoupling, so the number density falls like a^{-3}. If we assume that the main interaction is the annihilation of dark matter particles and antiparticles, we can write

$$\dot{n}_{\text{dm}} + 3Hn_{\text{dm}} = -\langle\sigma v\rangle \bar{n}_{\text{dm}} n_{\text{dm}} + \psi_{\text{dm}} \ , \tag{7.13}$$

where n_{dm} is the number density of the dark matter particles, $\bar{n}_{\text{dm}}$ is the antiparticle number density, ψ_{dm} is the rate of creation of the dark matter particles, and $\langle\rangle$ indicates average over the phase space distribution. Let us first consider the case when there is no particle-antiparticle asymmetry, so the chemical potential is zero, $\mu_{\text{dm}} = 0$. We will later see what happens if there is a conserved quantum number which enforces a particle-antiparticle asymmetry. (The term “thermal relic” is often used to refer only to the case when an asymmetry between particles and antiparticles is not important.) In equilibrium, equally many particles are being annihilated and created, so $\psi_{\text{dm}} = \langle\sigma v\rangle n_{\text{dm}}^2 \equiv \langle\sigma v\rangle n_{eq}^2 \equiv \Gamma n_{eq}$, where n_{eq} is the number density in equilibrium. Denoting the number of dark matter particles $N_{\text{dm}} \propto a^3 n_{\text{dm}}$ (and the equilibrium number by N_{eq}), we have

$$\frac{1}{N_{eq}} \frac{dN_{\text{dm}}}{d(\ln a)} = -\frac{\Gamma}{H}\left[\left(\frac{N_{\text{dm}}}{N_{eq}}\right)^2 - 1\right] \ . \tag{7.14}$$

In the limit $\Gamma \gg H$, interactions rapidly restore any deviations from the equilibrium distribution. If $N_{\text{dm}} > N_{eq}$, the right-hand side of (7.14) is negative, so the numbers will decrease, and the opposite for $N_{\text{dm}} < N_{eq}$. In the limit of weak coupling, $\Gamma \ll H$, we get $N_{\text{dm}} \approx$ constant. The time when the number of particles reaches this constant value is called decoupling (a term we already used with photons and neutrinos) or *freeze-out*. A crude approximation is to say that decoupling happens at exactly the temperature T_d where $H = \Gamma$, and that the number of particles follows the equilibrium behaviour before and is conserved afterwards, as we did for the neutrinos.

If a particle decouples while it is relativistic, the number density is of the order T_d^3. We calculated this starting from the phase space distribution, but it is fairly obvious, because T_d is the only relevant dimensional quantity. As we discussed above, such hot dark matter would have a large energy density today unless the mass is small. However, as a particle species becomes non-relativistic, the number density falls exponentially (still assuming that the chemical potential is zero), so the mass of the dark matter particle can be large while keeping the number density down.

The number density of a non-relativistic particle in thermal equilibrium (with zero chemical potential) at decoupling time t_d and temperature T_d is

$$n_{eq}(t_d) = g_{\rm dm} \left(\frac{m T_d}{2\pi} \right)^{3/2} e^{-m/T_d} \ , \tag{7.15}$$

where m is the mass of the dark matter particle. From this we get the density today as (assuming negligible decay)

$$n_{\rm dm}(t_0) = \frac{a(t_d)^3}{a(t_0)^3} n_{eq}(t_d) = \frac{g_{*S}(T_0)}{g_{*S}(T_d)} \left(\frac{T_0}{T_d} \right)^3 n_{eq}(t_d) \ , \tag{7.16}$$

where we have used the relation $g_{*S}(T) T^3 a^3 =$ constant, which follows from conservation of entropy. The total energy density is $\rho_{\rm dm} = m n_{\rm dm}$.

In order to determine the number density of a thermal relic, we need to know the mass, the decoupling temperature and the number of degrees of freedom at decoupling. At decoupling, $\Gamma = n_{eq}(t_d)\langle\sigma v\rangle$, so we need to know the mean of the cross section times the velocity. The cross-section depends on the details of the particle physics, but we can roughly parametrise the annihilation cross-section as $\sigma v \propto v^{2q}$, where $q = 0$ for annihilation in the ground state (s-wave), and $q = 1$ for annihilation in the p-wave state. This can be understood as an expansion in the square of the velocity, and since $v \ll 1$, only the leading term is relevant. (The p-wave term is only important if annihilation in the ground state is forbidden or strongly suppressed for some reason.) For a non-relativistic particle, $\langle|v|\rangle = \sqrt{8/\pi}\sqrt{T/m}$, so we write $\langle\sigma v\rangle = \sigma_0 (T/m)^q$. We therefore have

$$\Gamma(t_d) = \sigma_0 m^3 \frac{g_{\rm dm}}{(2\pi)^{3/2}} y^{-q-3/2} e^{-y} \ , \tag{7.17}$$

where we have defined $y \equiv m/T_d$; we have $y \gg 1$ since the dark matter particle is non-relativistic.

According to the Friedmann equation, the Hubble parameter is given by

$$3H^2 = 8\pi G_{\rm N} \frac{\pi^2}{30} g_*(T) T^4 = \frac{\pi^2}{30 M_{Pl}^2} g_*(T) T^4 \ , \tag{7.18}$$

so

$$H(t_d) = \pi \sqrt{\frac{g_*(T_d)}{90}} \frac{m^2}{M_{Pl}} y^{-2} \ . \tag{7.19}$$

Equating $\Gamma(t_d) = H(t_d)$, we get an equation from which we can solve the decoupling temperature in units of the dark matter mass, y,

$$N y^{1/2-q} e^{-y} = 1 \ , \tag{7.20}$$

where $N \equiv \sqrt{45/(4\pi^5 g_*(T_d))} g_{\rm dm} M_{Pl} m \sigma_0$. For a given value of $g_{\rm dm} m \sigma_0/\sqrt{g_*(T_d)}$, we can straightforwardly solve y numerically from (7.20). However, to get some analytical understanding, we can write (7.20) as

$$y = \ln N + \left(\frac{1}{2} - q\right) \ln y \ , \tag{7.21}$$

and solve iteratively, so that

$$\begin{aligned} y_0 &= \ln N \\ y_1 &= \ln N + \left(\frac{1}{2} - q\right) \ln(\ln N) \ , \end{aligned} \tag{7.22}$$

and so on; the first approximation will be good enough for us.

From (7.15) and (7.16), the relic abundance is

$$\begin{aligned} n_{\rm dm0} &= \frac{g_{*S}(T_0)}{g_{*S}(T_d)} \frac{g_{\rm dm}}{(2\pi)^{3/2}} y^{3/2} e^{-y} T_0^3 \\ &= \frac{g_{*S}(T_0)}{g_{*S}(T_d)} \frac{g_{\rm dm}}{(2\pi)^{3/2}} N^{-1} y^{1+q} T_0^3 \\ &= \frac{\pi^3}{\sqrt{360}} \frac{g_{*S}(T_0)}{\zeta(3)} \frac{y^{1+q}}{\sqrt{g_*(T_d)} M_{Pl} m \sigma_0} n_{\gamma 0} \\ &\approx 5.31 \frac{y^{1+q}}{\sqrt{g_*(T_d)} M_{Pl} m \sigma_0} n_{\gamma 0} \ , \end{aligned} \tag{7.23}$$

where we have used (7.20), put $g_{*S}(T_d) = g_*(T_d)$ (we assume that no particles are becoming non-relativistic as the dark matter decouples) and $g_{*S}(T_0) \approx 3.91$, and traded the temperature today for the photon number density via the relation $n_\gamma = 2\zeta(3)T^3/\pi^2$. The relic energy density $\rho_{\rm dm0} = m n_{\rm dm0}$ depends on the mass only logarithmically via y, apart from the possible mass dependence of σ_0.

7.7 The WIMP miracle

Let us consider a particle with with $g_{\rm dm} = 4$ (for example, a spin $\frac{1}{2}$ fermion with both left- and right-handed components), mass m not too different from GeV, weak-scale annihilation cross section $\sigma_0 \sim G_F^2 E^2 \sim G_F^2 m^2$, where the Fermi constant is $G_F \approx 1.17 \times 10^{-5}$ GeV^{-2}. Let us also assume that the particle annihilates via the s-wave process, $q = 0$. Then we have $n_{\rm dm0} \propto m^{-3}, \rho_{\rm dm0} \propto m^{-2}$. In the Standard Model, $g_*(T_d) = 75.75$ for 4 GeV $> T >$ 1 GeV, and let us adopt that value. We then have $N \approx 2.9 \times 10^7 (m/\text{GeV})^3$, or $\ln N \approx 17 + 3\ln(m/\text{ GeV})$, which is also the approximate the value of y. We thus get $T_d \approx m/[17 + 3\ln(m/\text{ GeV})]$. This is consistent with the adopted value of $g_*(T_d)$ only for roughly 40 GeV $\gtrsim m \gtrsim$ 10 GeV, but since $g_*(T_d)$ enters only logarithmically, the value of T_d is not sensitive to the precise number of degrees of freedom. These numbers give

$$\begin{aligned} n_{\rm dm0} &\approx 3\times10^{-8}\left(1+0.2\ln\frac{m}{\rm GeV}\right)\left(\frac{m}{\rm GeV}\right)^{-3} n_{\gamma0} \\ &= 3\times10^{-8}\eta^{-1}\left(1+0.2\ln\frac{m}{\rm GeV}\right)\left(\frac{m}{\rm GeV}\right)^{-3} n_{B0} \\ &\approx 50\left(1+0.2\ln\frac{m}{\rm GeV}\right)\left(\frac{m}{\rm GeV}\right)^{-3} n_{B0}\ , \end{aligned} \tag{7.24}$$

where we have taken $\eta = 6\times10^{-10}$. Since $m_b \approx 1$ GeV, we have

$$\rho_{dm0} \approx 50\left(1+0.2\ln\frac{m}{\rm GeV}\right)\left(\frac{m}{\rm GeV}\right)^{-2} \rho_{b0}\ . \tag{7.25}$$

For $m = 1$ GeV, we would have $\rho_{\rm dm0}/\rho_{b0} \approx 50$, whereas $m = 100$ GeV gives $\rho_{\rm dm0}/\rho_{b0} \approx 10^{-2}$. As $\rho_{b0} \approx 0.05\rho_{c0}$, we get the bound $m \gtrsim 2$ GeV on the mass of the dark matter particle in order for its present density not to exceed the critical density. This is called the *Lee–Weinberg* bound. We get the observed ratio $\rho_{\rm dm0}/\rho_{b0} \approx 6$ for $m \approx 3$ GeV. Note the assumptions in the derivation of the bound: the particle is assumed to be a thermal relic (i.e. the number density is determined by the thermal equilibrium distribution at decoupling) and the annihilation occurs via the s-wave process.

The fact that a thermal relic with weak cross section and a mass not too different (in logarithmic terms) from the weak scale gives the right relic abundance is called the *WIMP miracle*. However, in the MSSM, a weakly interacting dark matter particle with a mass of a few GeV would already have been detected in collider experiments. The lower mass limit from colliders experiments for fermionic SUSY partners in the MSSM is 15 GeV [7]. With upcoming LHC data, the limit is expected to rise. (Lighter particles can be viable in more complicated models, however.) The preferred range for dark matter masses is of the order 100 GeV or so in the usually studied models. One can still get the right relic abundance by making the self-annihilation cross section smaller so that more particles remain, and extensions of the Standard Model such as MSSM contain enough free parameters to adjust the cross sections and masses. However, they can be independently tested in colliders and via direct and indirect detection of the dark matter particles, which we will shortly discuss.

7.8 Asymmetric dark matter

It is noteworthy that the observed dark matter abundance is so close to the baryon abundance, given that in the scenario discussed above the two are determined by completely different physics. The baryon number density is determined by the conservation of baryon number after *baryogenesis* in the primordial universe, while for a WIMP thermal relic the dark matter number density is determined by the balance between weak interactions and gravity via the freeze-out temperature.

There are also models where the dark matter abundance is determined by a conserved quantum number, as is the case for baryons. It is illustrative to first consider what would happen if there were no baryon-antibaryon asymmetry. Then the baryon abundance would be determined by the freeze-out of nucleon annihilations just as in the case for WIMP dark matter. We have $g = 4$ (protons and neutrons both have 2 spin states) and $m_N = 0.94$ GeV. The nucleon-antinucleon annihilation cross section is $\langle\sigma v\rangle = \sigma_0 \sim m_{\pi^0}^{-2}$, where the neutral pion mass is $m_{\pi^0} = 0.135$ GeV. We take $g_*(T_d) = 10.75$, which is the value for T_d between 100 MeV and 0.5 MeV. These numbers give $N \approx 6 \times 10^{19}$, or $\ln N \approx 46$, which gives $y \approx 50$. For the freeze-out temperature we get $T_d \approx 19$ MeV. The resulting nucleon abundance is $n_{N0} \approx 7\times10^{-19} n_{\gamma 0}$, which is about 10^{-9} times smaller than the real nucleon density $n_{N0} = n_{B0} = 6 \times 10^{-10} n_{\gamma 0}$.

This failure of the reasoning based on the naive freeze-out argument which does not account for the presence of a conserved quantum number can be light-heartedly called the *"baryon catastrophe"*. The lesson is that primordial baryon asymmetry and the conservation of baryon number are essential in determining the baryon density.

We don't know what is the correct theory of particle physics that determines the dark matter density. In many models, such as MSSM, there is no dark matter-antimatter asymmetry. However, there are also models where the dark matter carries a conserved quantum number which has an asymmetry generated at early times. In particular, this is the case in some *technicolour* models. (An overview of asymmetric dark matter can be found in [8].)

In the Standard Model colour interaction, quarks are the relevant degrees of freedom at high energies, but at low energies they are bound into mesons and baryons. Technicolour is a higher energy version of the same idea. In technicolour, the Higgs is not an elementary particle, but a bound state of some elementary fields which become visible when probing sufficiently high energies. Technicolour models also contain other bound states, just like QCD does, and one of those bound states could be the dark matter particle. In correspondence to the baryon number B of the Standard Model, there is the *technibaryon number* T_B, carried by elementary technicolour particles and their bound states. If there is a conserved asymmetry in the technibaryon number, the abundance of dark matter particles may be determined by this asymmetry, and it can be very different from the freeze-out abundance we calculated above, just as with baryons.

If the process which generates the asymmetry in the dark matter is related to the process which generates the asymmetry in the baryons (*baryogenesis*), then the baryon and dark matter number densities are naturally related to each other. This possibility is called *cogenesis*. Alternatively, the quantum numbers could be related because they are mixed by some later process, a possibility called *sharing*. The details depend on the particle physics models, and as in the case of WIMP thermal relics, we keep the discussion at a general level.

If the dark matter particle carries one unit of the conserved quantum number Q (which could for example be the technibaryon number) and the symmetry-violating interactions produce N units of Q for every unit of B, and there is no mixing afterwards, the dark matter abundance today is simply

$$n_{\mathrm{dm}0} = N n_{B0} \ , \tag{7.26}$$

so

$$\frac{\rho_{\mathrm{dm}0}}{\rho_{B0}} = N \frac{m_{\mathrm{dm}}}{m_N} \ , \tag{7.27}$$

which agrees with the observed ratio ≈ 6 for $m_{\mathrm{dm}} = 6/N$ GeV.

One constraint on such models is that the phase space distribution of the dark matter particles has to correspond to CDM (or WDM). So the dark matter particle cannot have decoupled at the electroweak crossover with a thermal distribution function if its mass is smaller than 100 GeV. However, a model where the distribution function is not thermal would be possible – the essential thing is that the high momentum states of the dark matter particles are not occupied. From the point of view of technicolour models, $m_{\rm dm} \lesssim 10$ GeV is also an unnaturally low mass unless $N \ll 1$, since the technicolour scale has to be $\gtrsim 1$ TeV to be consistent with collider experiments (no technicolour bound states –or any other signatures of technicolour for that matter– have been observed). Naively, one would expect the mass of the stable technicolour dark matter particle to be of this order, or at least of the order of the Higgs mass, $m_H = 126$ GeV, since they have the same origin as bound states. But there could be a reason why the lightest stable fermionic bound state is much lighter than a bosonic unstable state. (In QCD, the lightest bosonic bound states, the pions with $m_{\pi^0} = 135$ GeV and $m_{\pi^\pm} = 140$ GeV, are about an order of magnitude lighter than the lightest stable bound state, the proton with $m_p = 938$ GeV, because of *chiral symmetry.*)

Alternatively, we can have reactions that mix particles carrying baryon number and particles carrying Q together, so that their relative abundance depends freeze-out temperature T_f of these interactions. Let's say that we have reactions which interconvert baryons and dark matter particles,

$$dm + X \leftrightarrow q + Y \ , \tag{7.28}$$

where q stands for a quark, which carries $B = 1/3$, dm stands for the dark matter particle which carries $Q = 1$ (or any other particle carrying the same quantum number), and X and Y are particles which carry neither B nor Q, and we assume we can neglect their chemical potentials. We then have, as long as these reactions are in equilibrium, $\mu_{\rm dm} = \mu_q$. Let us assume that these reactions freeze out at the electroweak crossover, and take the particle carrying the quantum number to be massless. (Since the top quark receives a mass of the order of the electroweak scale at the crossover, this assumption may seem questionable. However, at least in some technicolour models, the mass of the top quark does not make a difference [9].)

We assume that the technicolour particles are in thermal equilibrium. In order for them to count as CDM, we then need $m_{\rm dm} \gg T_f$. We thus have

$$\begin{aligned}
n_B - \bar{n}_B &= g_B T^3 \frac{\mu_B}{T} \\
n_{Q_{\rm dm}} - \bar{n}_{Q_{\rm dm}} &= g_{\rm dm} \left(\frac{m_{\rm dm} T}{2\pi}\right)^{3/2} e^{-\frac{m_{\rm dm}}{T}} \left(e^{\frac{\mu_{\rm dm}}{T}} - e^{-\frac{\mu_{\rm dm}}{T}}\right) \\
&\simeq 2\frac{\mu_{\rm dm}}{T} g_{\rm dm} \left(\frac{m_{\rm dm} T}{2\pi}\right)^{3/2} e^{-\frac{m_{\rm dm}}{T}} \ ,
\end{aligned} \tag{7.29}$$

where we have taken into account that the asymmetries and thus the chemical potentials are small, and $g_B = 24$ (the number of degrees of freedom in the quarks is 72, and each quark has $B = 1/3$). Note that just as g_B is the number of degrees of freedom which carry the conserved quantum number B which ends up in baryons, $g_{\rm dm}$ is the number of degrees of freedom which carry $Q_{\rm dm}$, which will in the late universe end up in the dark matter particles only. Equating the chemical potentials and noting that today $\rho_{B0} = m_N n_{B0}$, we obtain

$$\frac{\rho_{\rm dm0}}{\rho_{B0}} = \frac{g_{\rm dm}}{12(2\pi)^{3/2}} \frac{m_{\rm dm}}{m_N} \left(\frac{m_{\rm dm}}{T_f} \right)^{3/2} e^{-\frac{m_{\rm dm}}{T_f}} \ . \tag{7.30}$$

Taking $g_{\rm dm} = 100$ and $T_f = 160$ GeV, we get the observed abundance for $m_{\rm dm} \approx 1200$ GeV ~ 1 TeV. (Note that the temperature at which the electroweak crossover happens may change from the Standard Model value 160 GeV due to the new particles and interactions present in technicolour.)

7.9 Dark matter vs. modified gravity

Since all evidence for non-baryonic dark matter comes from its gravitational effects, it could in principle be possible to explain the observations by instead changing the law of gravity. Until the dark matter particle is detected, there is some room for uncertainty. The problem for such *modified gravity* proposals is that there are so many different observations explained by dark matter, in different physical systems: motions of stars in galaxies, motions of galaxies in clusters, gravitational lensing, large-scale structure, CMB anisotropies and so on [1, 10]. Gravity has to be adjusted in a different manner for these different observations, and the resulting models are rather contrived. Expressed another way, the dark matter scenario is very predictive: the simple hypothesis of a massive particle with weak couplings to itself and to the Standard Model particles explains a number of disparate observations and has made several successful predictions.

One example which got a lot of attention a few years ago is the *Bullet cluster* [11]. Shown in figure 3 is the collision between two clusters of galaxies. According to the dark matter scenario the mass of a galaxy cluster has three main components: 1) visible galaxies, 2) intergalactic gas and 3) cold dark matter. The last component is expected have the largest mass, and the first one the smallest. When two clusters of galaxies collide, it is unlikely for individual galaxies to crash, and the intergalactic gas is too thin to noticeably slow down the relatively compact galaxies. On the other hand, the intergalactic gas components do not travel through each other freely, but are slowed down and heated up by the collision. Thus after the clusters have passed through each other, much of the intergalactic gas is left behind between the receding clusters. Cold dark matter should be weakly interacting, and thus practically collisionless. Thus the CDM components of both clusters should also travel through each other unimpeded.

In the picture of the Bullet cluster, figure 3, the intergalactic gas has indeed been left behind the galaxies in the collision. The mass distribution of the system has been estimated from the gravitational lensing effect on the apparent shapes of galaxies behind the cluster. If there were no cold dark matter, most of the mass would be in the intergalactic gas, whose mass is estimated to be about five times that of the visible galaxies. Even in a modified gravity theory, we would expect most of the lensing effect to be where most of the mass is. However, expectation is not proof, so the observation cannot be said to rule out all possible models of modified gravity. Nevertheless, it does provide an example of a successful complex prediction of the cold dark matter hypothesis.

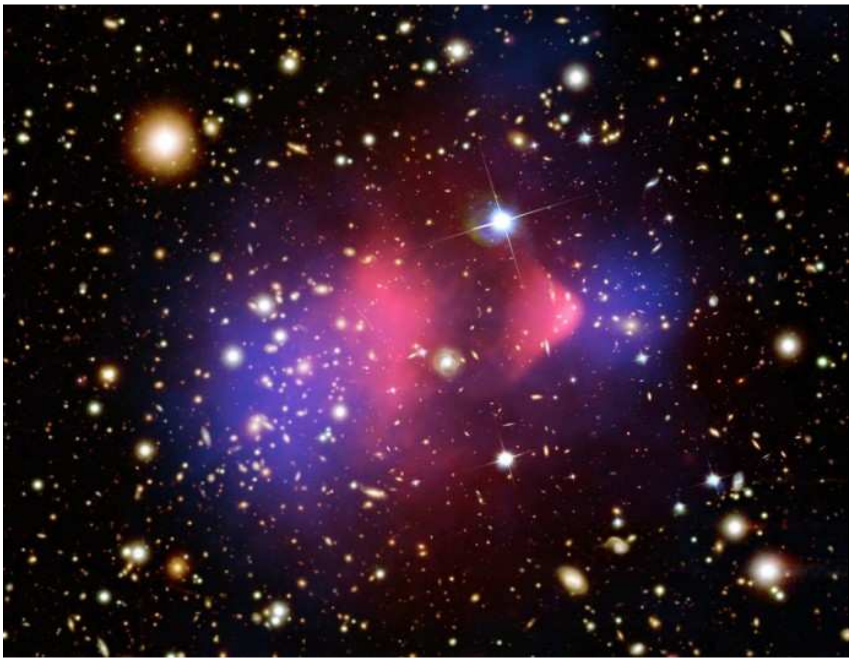

Figure 3: A composite image of galaxy cluster 1E 0657-56, also called the Bullet Cluster. It consists of two subclusters, a larger one on the left, and a smaller one on the right. They have recently collided and travelled through each other. One component of the image is an optical image which shows the visible galaxies. Superposed on it, in red, is an X-ray image, which shows the heated intergalactic gas, that has been slowed down by the collision and left behind the galaxy components of the clusters. The blue colour is another superposed image, which represent an estimate of the total mass distribution of the cluster, based on gravitational lensing. NASA Astronomy Picture of the Day 2006 August 24. Composite Credit: X-Ray: NASA/CXC/CfA/M. Markevitch et al. Lensing map: NASA/STScI; ESO WFI; Magellan/U. Arizona/D. Clowe et al. Optical: NASA/STScI; Magellan/U. Arizona/D. Clowe et al..

7.10 Direct detection

As we have seen, there are different plausible mechanisms for producing the observed dark matter abundance. (There also mechanisms that we did not discuss, involving neither a conserved quantum number nor a thermal relic, such as those relevant for axions and gravitinos.) These mechanisms are in turn realised in many different models. In order to distinguish between the models and confirm the identity of the dark matter particle, as well as to be sure that the correct interpretation of observations is really dark matter and not modified gravity, we need to observe dark matter via non-gravitational interactions.

Usually, detection of dark matter is divided into three different categories: producing the dark matter particle at colliders (*collider detection*), measuring the interactions dark matter with baryonic matter in the laboratory (*direct detection*) and measuring the end products of astrophysical dark matter annihilation or decay (*indirect detection*). A fourth category could be added, detecting the influence of dark matter on the evolution of stars and the intergalactic medium. For example, dark matter annihilation in the early universe can heat up the gas that forms stars and thus have an impact on the formation of early stars and reionisation. It has also been suggested that the first stars would be powered mainly by dark matter annihilation instead of fusion reactions; these have been dubbed 'dark stars' [12] (this is something of a misnomer, as they do shine!). Discussing such details requires delving into details of astrophysics, so we just mention the possibility. Detailed collider signals are also properly the topic of a specialised particle physics course, we simply note that if dark matter physics is related to the electroweak scale, whether via supersymmetry, technicolour or some other theory, then it is expected to be accessible in experiments at the LHC. On the other hand, axions or light warm dark matter candidates would not necessarily have any signature in high-energy colliders.

Let us first consider direct detection. Since dark matter is everywhere, including on (and in) the Earth, we should be able to see its interactions with baryonic matter if we look closely enough. As dark matter interactions with ordinary matter have to be weak in order to agree with cosmological observations, sensitive dedicated experiments are required. Mostly WIMPs, like neutrinos, pass through the Earth without interacting at all, but sometimes they interact with ordinary matter. A typical WIMP direct detection setup is a well isolated crystal or liquid sample, which is being observed to find the energy and momentum deposited inside it by a collision of a nucleus with a dark matter particle[5]. The problem is that there are many "background" events that may cause a similar signal: WIMP detectors see spurious signals all the time. Therefore one looks for an annual modulation in the signal. WIMPs, if they exist, have a particular velocity distribution related to the gravitational well of our galaxy. The simplest possibility is that they are, on average, at rest with respect to the Galactic rest frame. The Earth is moving with respect to this frame, because the Sun orbits the center of the Galaxy and the Earth orbits the sun. The annual change in the direction of Earth's motion should result in a corresponding variation in the detection rate.

[5]For dark matter particles which do not feel the weak interaction, different detection methods are needed. For axions, one kind of a detector is a low noise microwave cavity with a large magnetic field. An axion may interact with the magnetic field and convert into a microwave photon. No axions have so far been detected. Some dark matter candidates, such as gravitinos, may interact so weakly that they can in practice be detected only via their gravitational effect.

Let us estimate the expected energy deposition from the elastic collision of a dark matter particle and a nucleus. Dark matter velocities are non-relativistic (by definition), so in the laboratory frame we have from conservation of energy and momentum

$$\begin{aligned} mv^2 &= mv_{\rm dm}^2 + m_t v_t^2 \\ mv &= mv_{\rm dm} + m_t v_t \ , \end{aligned} \tag{7.31}$$

where m_t is the mass of the target nucleus, and m is still the dark matter mass. As the kinetic energy $\frac{1}{2}m_t v_t^2$ given to the nucleus, we get

$$\begin{aligned} E &= \frac{2m_t}{(1+m_t/m)^2} v^2 \\ &\approx \frac{2A}{(1+Am_N/m)^2} \left(\frac{v}{300\text{km/s}} \right)^2 \text{ keV} \ , \end{aligned} \tag{7.32}$$

where A is the mass number of the target nucleus and $m_t \approx Am_N$.

The velocity distribution of the dark matter particles is often taken to be Maxwellian (with a cut-off at the Galactic escape velocity), with a dispersion of $220/\sqrt{2}$ km/s, the velocity of the Solar system with respect to the Galaxy is 230 km/s, and the velocity of the Earth relative to the Sun is 30 km/s. A rough estimate of the typical root mean square velocity is thus $v \approx 300$ km/s. Note that the interaction strength is irrelevant for the energy exchange, it only affects the probability of the interaction (i.e. the rate of events observed in the detector). The expected annual modulation is roughly 30 (km/s)/v, which in our approximation is about 10%. There are uncertainties in the dark matter distribution and the rotation of the Solar system in the Galaxy, and the annual modulation rate can be between 1% and 10% [13].

The event rate depends on the dark matter-nucleus cross-section, $\sigma_{\rm dm-nucleus} \approx A^2\sigma_{{\rm dm}-p}$, where $\sigma_{{\rm dm}-p}$ is the dark matter-proton cross section. The dark matter-proton cross section can be completely different from the dark matter-dark matter annihilation cross section. The total number of events per unit time is given by the interaction rate of a single nucleus the number of nuclei in the target with mass M, which we denote by $N = M/(Am_N)$:

$$\begin{aligned} \Gamma N &= \langle \sigma_{\rm dm-nucleus} v \rangle n_{\rm dm} N \\ &\approx \frac{2\times 10^4 A}{\rm yr} \frac{M}{\rm ton} \frac{\langle \sigma_{{\rm dm}-p} v\rangle}{10^{-40}\text{cm}^2 \times 300\text{km/s}} \frac{\rho_{\rm dm}}{0.3 \text{ GeV/cm}^3} \left(\frac{m}{\rm GeV} \right)^{-1} , \end{aligned} \tag{7.33}$$

where we have put in typical values for the cross section, velocity and WIMP density. The latter two are determined by taking a given density profile for the dark matter as a function of radius and using the observed rotation curves, and they also agree with typical values obtained from galactic simulations of dark matter[6]. For comparison, the weak interaction annihilation cross section for 1 GeV mass is $\sigma \sim G_F^2$ GeV$^2 \sim 10^{-10}$ GeV$^{-2} \approx 4\times 10^{-38}$ cm$^2 \approx 10^{-27}$ cm^3/s, using the relation 197 MeV $\approx$ 1/fm.

[6]The energy density one gets in detailed analyses typically does not vary from $\rho_{\rm dm} = 0.3$ GeV/cm^3 by more than a factor of a few. However, strictly speaking, observations are consistent with no dark matter in the Solar system. The direct upper limit on the density of dark matter in the Solar system comes from the fact that no disruption of planetary orbits in has been observed, and it is about 10^6 times this value. As far as the Galactic rotation curves is concerned, dark matter mostly needed in the outer parts of the galaxy, though it is preferred also in our location.

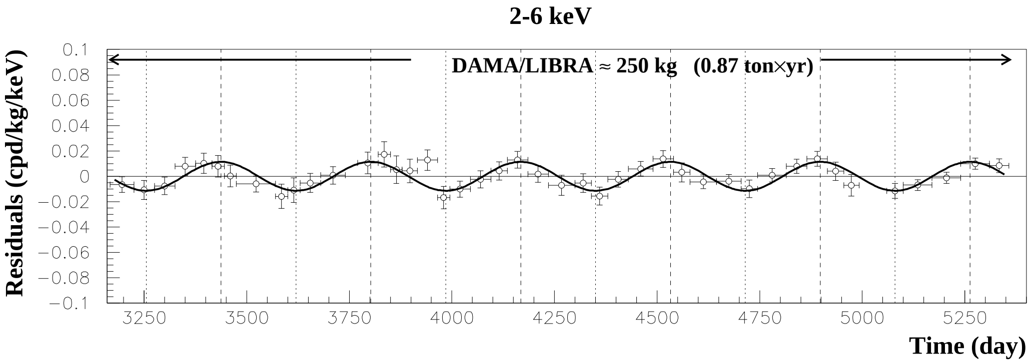

Figure 4: Modulation of the detection rate of the DAMA/LIBRA experiment in the 2-6 keV energy range, in units of counts per day/kg/keV. From [14].

One direct detection experiment, DAMA/LIBRA[7] claims to have detected dark matter. They see an annual modulation signal with the expected time of maximum rate (given the direction of the Solar system's velocity with respect to the galaxy and the direction of Earth's velocity with respect to the Sun). They use a sodium crystal with $A = 23$. The modulation of the rate is shown in figure 4. The peak of the energy is at 3 keV, corresponding to a dark matter particle mass of around 10 GeV. They had a total of about 1.17 ton×year of exposure in the beginning of 2010 (when figure 4 was released), so we would expect about $4 \times 10^5 \times \sigma_{\mathrm{dm}p}/(10^{-40}\mathrm{cm}^2)$ events, or about 0.1 events per day per kilogram. With a modulation rate of 10%, we would have a change of 0.01 events per day. This roughly agrees with the number 0.02 in figure 4 for $\sigma_{\mathrm{dm}p} \approx 10^{-40}\mathrm{cm}^2$. (Note that the y-axis for counts per day/kg/keV. We should integrate that number over the energy-dependent count rate over the range 2–6 keV to compare to our estimate; this will give a factor of order unity.) The CoGeNT experiment has also seen annual modulation. However, other direct detection experiments have ruled out ordinary WIMPs as an explanation for DAMA or CoGeNT, as shown in figure 5, and the interpretation of the data remains controversial. (For a bit more details on the various experiments, see [16].) Particle physics possibilities include non-elastic collisions involving an excited state of the dark matter particle, but systematic effects in the experimental setup are widely considered the most likely explanation. In any case, there are several direct detection experiments taking data, and if the dark matter is composed of standard WIMPs, it is possible they will be observed in the near future.

7.11 Indirect detection

Indirect detection refers to the case when the dark matter particle is identified through its annihilation or decay products. If there are no dark matter antiparticles around, as is the case for asymmetric dark matter, there is no annihilation signal. If the particle is stable or has a lifetime much longer than the age of the universe, there is no detectable signal from decays. We consider only annihilation.

The relic density of a thermal relic WIMP is determined by when the annihilation reactions freeze out (essentially because the density gets so low that particles and antiparticles don't meet). However, the density in local clumps grows during

[7]http://www.lngs.infn.it/lngs/htexts/dama/

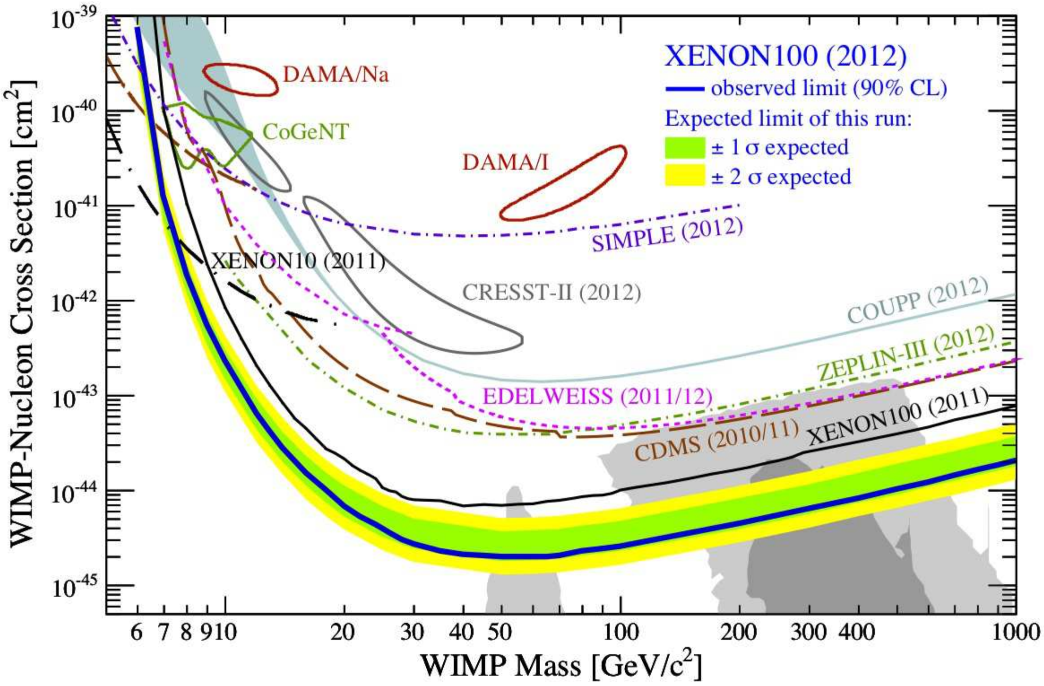

Figure 5: Allowed regions of parameter space for a WIMP interpretation of DAMA and CoGeNT results. The DAMA and CoGent regions are ruled out by other experiments. From [15].

structure formation, and this can lead to observable amounts of annihilation. (Note the similarity to nuclear reactions: they freeze out in the early universe, but light up again in regions where the density of baryonic matter rises sufficiently due to gravitational collapse.)

The amount of annihilation is proportional to the square of the dark matter density, so the largest signal is expected from regions with high dark matter density, such as dwarf galaxies or the centre of our own galaxy. Dark matter can also accumulate in the Sun and at the centre of the Earth, and though the numbers are much smaller, these locations are much nearer to us, so the detection is easier. However, only neutrinos can escape from the Sun or the centre of the Earth, whereas in the case of astrophysical objects we can observe several kinds of annihilation products – though there too the propagation of charged particles is a bit complicated. From the direction where we measure a positron or an antiproton we cannot deduce where the source is, since their paths are twisted by magnetic fields on the way. Therefore, only the detected number of charged particles carried useful information, not their direction (and even to calculate the expected numbers we have to make some assumptions about propagation). In contrast, photons at the relevant energies travel basically unimpeded through the galaxy, so we can immediately determine where they have come from. (Scattering of light due to dust is negligible at high energies.)

Let us consider the annihilation signal from the centre of our galaxy. The annihilation rate per particle is $\Gamma = \langle\sigma v\rangle n_{\rm dm}$, so the number of annihilation events per unit volume per unit time is $\langle\sigma v\rangle n_{\rm dm}^2 = \langle\sigma v\rangle m^{-2}\rho_{\rm dm}^2$. Integrating along the line of sight, the observed flux from annihilations into particle X is

$$\begin{aligned}\Phi_X(E) &= \frac{N_X(E)\langle\sigma v\rangle}{4\pi m^2}\frac{1}{\Delta\Omega}\int_{\Delta\Omega} d\Omega \int dl \rho_{\rm dm}^2 \\ &\approx 2\times 10^{-8}\frac{N_X\langle\sigma v\rangle}{10^{-30}{\rm cm}^3 s^{-1}}\left(\frac{\rm GeV}{m}\right)^2 \bar{J}(\Delta\Omega){\rm m}^{-2}{\rm s}^{-1}{\rm sr}^{-1}\ , \qquad (7.34)\end{aligned}$$

where $N_X(E)$ is the number of X particles of energy E produced in each annihilation, $\Delta\Omega$ is observed angular element and the integral $\int dl$ is over the line of sight, averaged over the angle. The reference value $10^{-30}{\rm cm}^3 s^{-1}$ is the weak scale annihilation cross section (with $m = 1$ GeV) times 300 km/s. The function $\bar{J}(\Delta\Omega)$ (the overbar stands for angular average) contains the uncertainties due to the dark matter distribution, and is defined as

$$\bar{J}(\Delta\Omega) \equiv \frac{1}{8.5{\rm kpc}}\left(\frac{1}{0.3\ {\rm GeV/cm}^3}\right)^2 \frac{1}{\Delta\Omega}\int_{\Delta\Omega} d\Omega \int dl \rho_{\rm dm}^2\ . \qquad (7.35)$$

The virtue of indirect detection is that the relevant cross section is the same one that determines the relic density (for thermal relics), unlike in direct detection, though the issue is complicated by model-dependent decay channels. However, the dark matter density profile at the galactic centre (and on small scales in general) is poorly known. In fact the mismatch between observations and simulations as regards the centres of galaxies is considered to be one of the most pressing issues of the CDM model. Simulations predict a sharper increase of density near the centre than inferred from observations of galaxies. For different choices of the density profile, one gets a range $\bar{J} \approx 30\ldots 10^6$ for $\Delta\Omega = 10^{-3}$ sr and $\bar{J} \approx 30\ldots 10^8$ for $\Delta\Omega = 10^{-5}$ sr [17]. The relevant angular size depends on the angular resolution of the instrument. In any case, for weak scale annihilation cross-sections, the expected flux is quite small, though not completely out of reach.

Observational limits from the flux of photons measured by the Fermi-LAT satellite on dark matter in a constrained version of the MSSM are shown are shown in figure 6.

There have recently been some observational signals that have been interpreted as evidence for dark matter. In particular, an excess of positrons was seen by the PAMELA satellite experiment in 2008, and confirmed by the Fermi satellite experiment in 2011. However, the rate is too high by a factor of about 10^3 if the annihilation cross section is taken to be fixed by the observed relic density, so the observations are inconsistent with the simple WIMP picture. Different options, from increased clumping of the dark matter to a mechanism which boosts the annihilation cross section at small velocities (i.e. today but not at decoupling) were suggested, but the interpretation remains uncertain. It is possible that the positron excess is due to astrophysical sources such as pulsars or supernova remnants. Another possible contributing factor is the generation of more positrons than expected by the scattering of other cosmic rays, as the details of cosmic ray propagation in the Galaxy are not clear.

One interesting indirect detection channel is neutrinos. High-energy neutrinos from outside the Solar system have been detected in 2010–2012 by the IceCube detector on the South Pole [19]. They could be from astrophysical sources, but an interpretation in terms of dark matter decay has also been proposed.

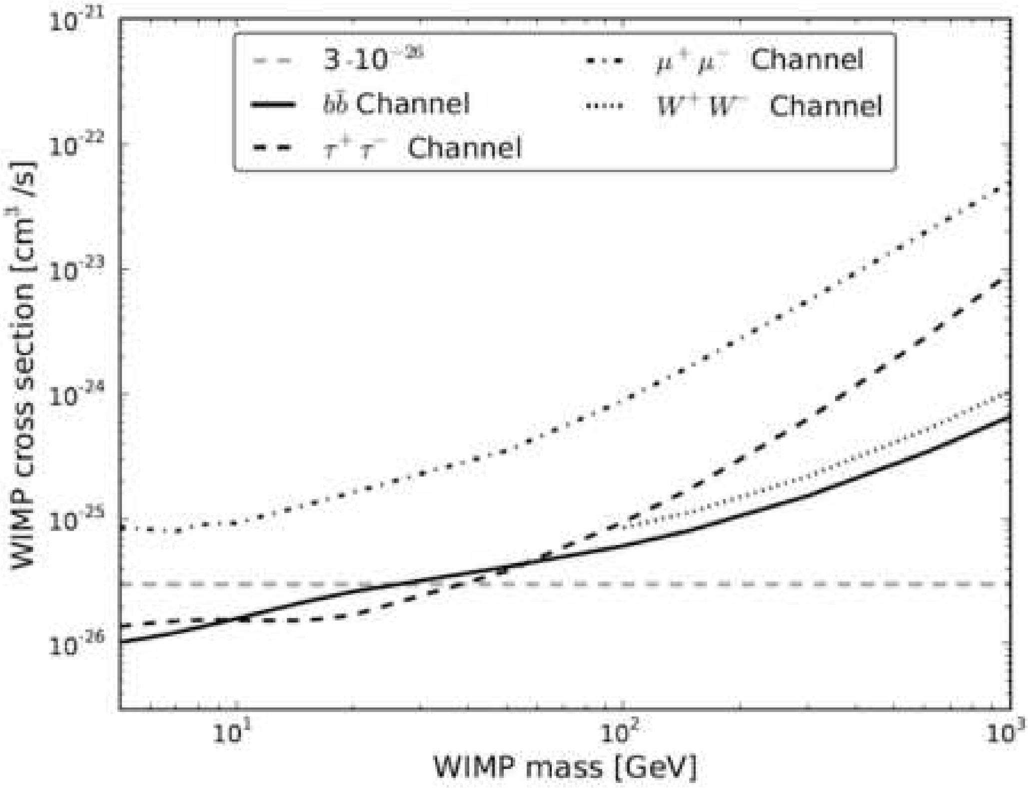

Figure 6: Observational limits on WIMP dark matter in a constrained version of the MSSM from dwarf spheroidal galaxies. From [18].

In April 2012, a monochromatic (i.e. occurring at one energy only) gamma ray signal of 130 GeV was reported from the galactic centre using the Fermi satellite data [20]. If confirmed, this would be a strong indication of dark matter, since astrophysical sources typically generate a continuum of energies, and dark matter annihilation is the only known source for emission at a single energy. However, in dark matter models, we also expect a continuum to accompany the monochromatic signal, since the dark matter decays also to other particles than photons, and some of these then decay to photons, producing a wide range of energies in the photon final states. Typically, the continuum signal is expected to be about 1000 times stronger than the monochromatic line (remember that dark matter couples weakly to photons!), and no such excess in the continuum emission is seen. So if the signal is due to dark matter, it has properties rather different from what is expected. At the moment, the matter remains open. (For more details on the various experiments, see [16].)

In summary, as with direct detection, the reach of indirect detection experiments is increasing, and many avenues are being investigated. Whether dark matter will be detected (via its non-gravitational interactions) depends on which model of dark matter is correct. It is worth bearing in mind that there are some candidate, such as gravitinos, whose non-gravitational interactions are too weak to detect in the foreseeable future.

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