COSMOLOGY I Homework 3 Fall 2015

Due on Monday September 21 by 14.15.

1. **The energy continuity equation.** Derive the equation

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$$

from the Friedmann equations. What does this equation say about conservation of energy? How does it relate to thermodynamics?

2. **Age of the closed universe.** Find the age-redshift relation for a closed universe ($K > 0$) with only matter (no radiation or vacuum energy). Calculate t_0 (the present age of the universe) if $H_0 = 70$ km/s/Mpc and a) $\Omega_0 = 1.1$, b) $\Omega_0 = 2$.

3. **The Einstein-de Sitter model.** Consider the spatially flat model with only matter, $\Omega_m = 1$.

a) Calculate the scale factor $a(t)$, the age-redshift relationship $t(z)$ and the angular diameter distance $d_A(z)$. (Express the age and the distances in units of the Hubble time H_0^{-1}.)

b) What is the particle horizon?

c) What is the age of the universe (in years) today and at $z = 1090$ if $h = 0.7$?

d) What is the angular diameter distance (in Mpc) to redshift $z = 1090$ if $h = 0.7$?

e) The function $d_A(z)$ has a maximum. At which redshift is it?

4. **The concordance model.** Suppose that we have $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$ and $\Omega_{\Lambda 0} = 0.7$, so that $\Omega = \Omega_m + \Omega_\Lambda = 1$ and the universe is spatially flat.

a) Find the age of the universe today and at redshift $z = 1090$.

b) When is the matter density equal to the vacuum energy density? (Give both t and z.)

c) The scale factor has an inflection point, where $\ddot{a} = 0$, at which the expansion starts to accelerate. When does this happen, in t and in z?

(Hint: Use the substitution $x^{3/2} = b \sinh \phi$ for the integral $\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}$.)