COSMOLOGY I Homework 5 Fall 2015

Due on Monday October 5 by 14.15.

1. **Redshift of non-relativistic particles.** Consider a distribution of non-interacting non-relativistic particles in kinetic equilibrium in an expanding universe. Using the fact that momentum redshifts as $p_2 = (a_1/a_2)p_1$, show that the distribution remains in kinetic equilibrium. How are the temperature and the chemical potential redshifted?

2. **Time and distance scales of the early universe.** Let t denote the age of the universe, t_H the Hubble time, d_{hor} the horizon distance, $d_{pou}(t) = a(t)d_{hor}(t_0)$ the size of the presently observable universe at time t, n_B the baryon density and S the total entropy of the region inside the present-day horizon. Find the values of these quantities at $T = 100$ GeV and 1 MeV. Compare to the density of water and air on Earth and to the density of a nucleus. (Calculate the horizon distance numerically, assuming the spatially flat ΛCDM model with $\Omega_{\Lambda 0} = 0.7$ and $h = 0.7$.)

3. **Transparency of the universe.** We say that the universe is transparent when the photon mean free path λ_γ is larger than the Hubble length $l_H = H^{-1}$, and opaque when $\lambda_\gamma < l_H$. The photon mean free path is determined mainly by the scattering of photons by free electrons, so that $\lambda_\gamma = 1/(\sigma_T n_e)$, where $n_e = x n_e^*$ is the number density of free electrons, n_e^* is the total number density of electrons, and x is the ionization fraction. The cross section for photon-electron scattering is independent of energy for $E_\gamma \ll m_e$ and is then called the Thomson cross section, $\sigma_T = \frac{8\pi}{3}(\alpha/m_e)^2$, where α is the fine-structure constant. In recombination x falls from 1 to 10^{-4}. Assume instantaneous recombination at $1 + z = 1300$.

   a) Show that the universe is opaque before recombination and transparent after recombination. (You can assume a matter-dominated universe; see below for parameter values.)

   b) Interstellar baryonic matter gets later reionized (to $x \sim 1$) by light from the first stars. What is the earliest redshift when this can happen without making the universe opaque again? (You can assume that most ($\sim$ all) matter has remained interstellar.)

   Calculate for $\Omega_{m0} = 1.0$ and $\Omega_{m0} = 0.3$ (note that Ω_m includes nonbaryonic matter). Use $\Omega_\Lambda = 0$, $h = 0.7$ and $\eta = 6 \times 10^{-10}$.

4. **Electron density in the early universe.**

   a) Assume $T \ll m_e$ and $|\mu_e| \ll m_e$. Give n_e in terms of n_p^* and T (i.e. get rid of the chemical potential). Here $n_e \equiv n_{e^+} + n_{e^-}$ and $n_p^* \equiv n_p + n_H + 2n_{He} + \ldots$ (i.e. n_p^* includes all protons, whether bound or free). Hint: at these temperatures there are essentially no antiprotons, the only charged particles are protons, electrons and positrons, and the total charge is zero.

   b) Ignore neutrons, so that $n_p^* = n_B$, and use $\eta = 6 \times 10^{-10}$. Find n_{e^+}/n_{e^-} and μ_e at $T =$ 50 keV and 10 keV. At what temperatures is $|\mu_e| \ll m_e$ valid? You can keep using the Maxwell-Boltzmann approximation (why?).