COSMOLOGY I Homework 6 Fall 2015

Due on Monday October 12 by 14.15.
This is the last problem set of Cosmology I.

1. **Tremaine–Gunn limit.** Neutrinos are non-relativistic today. Therefore the cosmic "neutrino gas" is not homogeneous, but some of the neutrinos have fallen into galaxies. Let us suppose they dominate the mass of galaxies (i.e. ignore other forms of matter). We know the mass of a galaxy (within a certain radius) from its rotation velocity. The mass could come from a small number of heavier neutrinos or a large number of lighter neutrinos, but the available phase space (you don't have to assume a thermal distribution) limits the total number of neutrinos whose velocity is below the escape velocity. This leads to a lower limit of the neutrino mass m_ν. Let r be the radius of the galaxy and v its rotation velocity at this distance.

   a) Find the minimum m_ν needed for neutrinos to dominate the galaxy mass, assuming that all three species of neutrinos have the same mass. A rough estimate is enough: you can e.g. assume that the neutrino distribution is spherically symmetric, and that the escape velocity within radius r equals the escape velocity at r. Give the numerical value for the case $v = 200$ km/s and $r = 10$ kpc.

   b) Repeat the calculation assuming that only ν_τ is massive. (In reality, we know that at least two of the neutrinos have non-zero masses.)

   Today we know that neutrinos are only a small part of dark matter, but the reasoning applies to any fermions.

2. **Neutrino chemical potential.** In the standard analysis, neutrino chemical potential is assumed to be negligible. Neutrino chemical potential affects BBN in two ways: 1) The energy density of neutrinos is larger, $\rho + \bar\rho = \frac{7}{8} g \frac{\pi^2}{15} T^4 \left(1 + \frac{30}{7\pi^2}\left(\frac{\mu}{T}\right)^2 + \frac{15}{7\pi^4}\left(\frac{\mu}{T}\right)^4\right)$, leading to faster expansion of the universe. 2) The chemical potential of the electron neutrino is reflected in the neutron and proton chemical potentials and thus in the n/p ratio. The chemical potential redshifts as the neutrino temperature. Define $\xi_i \equiv \mu_{\nu_i}/T_\nu$. Suppose that we have $\xi_e = 0.056$ (the current upper limit), $\xi_\mu = \xi_\tau = 0$. Use a simplified nucleosynthesis model where the reactions $n + \nu_e \leftrightarrow p + e^-$, etc. are in equilibrium down to $T = 0.8$ MeV, when the neutrinos decouple instantaneously, after which neutrons decay (into protons) with a half-life $t_{1/2} = 610$ s, and nucleosynthesis, where all neutrons go into ^{4}He nuclei, happens instantaneously at $T = 70$ keV. Use the $T \propto t^{-1/2}$ expansion law, which applies after electron annihilation, for the whole period. How much ^{4}He is produced in this case? Is it more or less than in the standard ($\xi_e = 0$) case?

3. **WIMP miracle.** Consider a WIMP that has a constant thermally averaged self-annihilation cross section times velocity, $\langle \sigma v \rangle = \sigma_0$, mass m and two internal degrees of freedom. Assume that the WIMP decouples instantaneously when $\Gamma = H$, that $m \gg T_d$ and $g_*(T_d) = g_{*S}(T_d) = 100$, and that $\eta = 6 \times 10^{-10}$. Find the dark matter energy density today ρ_{dm0} relative to the baryon energy density today ρ_{b0}, as a function of σ_0 and m. Give the numerical value for $\sigma_0 = 10^{-38}$ cm^2 and $m = 100$ GeV.

4. **Baryon catastrophe.** Consider a universe with zero baryon number. In analogy with the WIMP calculation above, find the present value of the energy density of nucleons and antinucleons left over from annihilation, relative to photons. Use $g = 4$, $m_N = 0.94$ GeV, $\langle \sigma v \rangle = m_{\pi^0}^{-2}$ with $m_{\pi^0} = 0.135$ GeV and $g_*(T_d) = 10.75$.