COSMOLOGY II Homework 7 Fall 2015

Due on Monday November 2 by 12.15.

1. **The oldness problem.** Assume that the universe contains only radiation and $g_* = 100$. Suppose that at the Planck density $\rho = M_{\rm Pl}^4$ the density parameter was less than one, and not extremely close to one, say $\Omega = 0.9$.

   a) What is the age of the universe when Ω becomes smaller than 10^{-2}?
   b) What is the age of the universe when the temperature falls below $T = 2.7$ K?

2. **Spatial curvature.** Assume that at the beginning of inflation we have $|\Omega_K| = 0.1$.

   a) Calculate, as a function of the reheating temperature $T_{\rm reh}$, how many e-folds of inflation are required to reduce present-day spatial curvature to $|\Omega_{K0}| < 10^{-2}$. (Assume $h = 0.7$ and that neutrinos are massless.) Approximate that the expansion rate at the beginning of inflation is completely dominated by the inflaton, that the inflaton field value does not change during inflation and that reheating happens instantaneously.

   b) In which directions do the above approximations change the result?

   c) What is the number of e-folds for $T_{\rm reh} = 10^7$ GeV?

3. **Slow-roll parameters.** Demonstrate that $\varepsilon(\varphi) \ll 1$ and $|\eta(\varphi)| \ll 1$ are necessary conditions for the slow-roll approximation to be valid. Why are these conditions not sufficient?