COSMOLOGY II Homework 8 Fall 2015

Due on Monday November 9 by 12.15.

1. **Small-field inflation.** Suppose that the inflaton potential is

$$V(\varphi) \approx V_0 \left[1 - \lambda \left(\frac{\varphi}{M_{\text{Pl}}} \right)^4 + \ldots \right] .$$

The omitted terms (which are responsible for keeping $V(\varphi)$ nonnegative) are assumed to be negligible in the region of interest (the slow-roll section). Assume further that

$$\lambda \left(\frac{\varphi}{M_{\text{Pl}}} \right)^4 \ll 1$$

in the slow-roll section, so that we can approximate $V(\varphi) \approx V_0$, except when calculating derivatives. a) Find φ_{end} from the condition $|\eta| = 1$.
b) Find $N(\varphi)$. (When φ is sufficiently deep in the slow-roll section, so that $|\eta| \ll 1$, you can make an approximation where the term that depends on φ_{end} is dropped from $N(\varphi)$. Justify this.)
c) Find φ, $\eta(\varphi)$, and $\varepsilon(\varphi)$ in terms of N and give the values for $N(\varphi) = 50$.
d) Find how much the value of φ changes during the last 50 e-folds of inflation.
e) Show from the above assumptions that the condition $\varepsilon \ll |\eta|$ holds in the slow-roll region.

2. **The linear equations.** Starting from eqs. (9.12) and (9.14), derive eqs. (9.22) and (9.23).
a) Show that if $w = c_s^2$, then $\delta_{\boldsymbol{k}} = -2\Phi_{\boldsymbol{k}} =$ constant is a solution in the long-wavelength limit $k \ll aH$. (You may assume that $a \propto t^n$ with $n > 1/4$ and that $c_s^2 =$ constant ≥ 0.)
b) Is this the only solution? If so, explain why. If not, is it the dominant solution at late times?

3. **Gaussian random variables.** Show that if α is a real Gaussian random variable with $\langle \alpha \rangle = 0$, then

$$\langle |\alpha|^4 \rangle = 3 \langle |\alpha|^2 \rangle^2 ,$$

and that, if a is a complex Gaussian random variable (real and imaginary parts independent of each other), with $\langle a \rangle = 0$, then

$$\langle |a|^4 \rangle = 2 \langle |a|^2 \rangle^2 .$$