**COSMOLOGY II** Homework 11 Fall 2015

Due on Monday November 30 by 12.15.

1. **Matter perturbations in the dark energy dominated era.** If dark energy is vacuum energy, it will eventually become completely dominant and the universe will expand exponentially. Find how matter density perturbations behave then on scales for which $k \gg aH$ (take $c_s^2 \approx v^2 \approx 0$). (Note that the total density contrast and the matter density contrast are different, and vacuum energy density has no perturbations.)

2. **Power spectrum and the mean square density fluctuation.** Starting from the definition $\sigma^2(R) \equiv \langle \delta(\boldsymbol{x}, R)^2 \rangle$, show that if the spectrum of density perturbations follows a power law,
$$\mathcal{P}_\delta(k) = A k^{n+3} \ ,$$
then the mean square density fluctuation at scale $R$ is
$$\sigma^2(R) = \frac{1}{2}\Gamma\left(\frac{n+3}{2}\right)\mathcal{P}_\delta(R^{-1}) \ .$$
Use the Gaussian window function, $W(x/R) = \exp(-x^2/(2R^2))$. (The convolution theorem may be helpful.)

3. **Timescales of structure formation.** Structures on scale $R$ typically form when $\sigma^2(R) = 1$. What is the age and redshift of the universe when structures with size $R$ form, when $R$ is a) pc, b) Mpc, c) 10 Mpc, d) $k_{\rm eq}^{-1}$, e) 1000 Mpc?

   Assume that the universe is spatially flat and matter-dominated. Take the primordial spectrum of comoving curvature perturbations to be scale-invariant with the amplitude $A = 5 \times 10^{-5}$. For the transfer function, you can use the approximation
$$T(k)^2 = \frac{1}{1 + \beta(k/k_{\rm eq})^4} \ ,$$
   with $\beta = 3 \times 10^{-4}$. As the window function, you can use the step function $W(R) = \theta(R^{-1} - k)$ instead of a Gaussian. Assume $t_{\rm eq} = 50\,000$ years and $k_{\rm eq}^{-1} = 100$ Mpc.