

$$1 \text{ eV} = 11600 \text{ K} = 1.60 \times 10^{-19} \text{ J} = 5.07 \times 10^6 \text{ m}^{-1} = 1.52 \times 10^{15} \text{ s}^{-1} = 1.78 \times 10^{-36} \text{ kg}$$

$$c = 1 = 3.00 \times 10^8 \text{ m/s}$$

$$\hbar = 1 = 197 \text{ MeVfm}$$

$$1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$$

$$1 \text{ a} = 3.156 \times 10^7 \text{ s}$$

$$h \equiv H_0 / (100 \text{ km/s/Mpc})$$

$$(100 \text{ km/s/Mpc})^{-1} = 9.78 \times 10^9 \text{ a}$$

$$= 2998 \text{ Mpc} = 4.69 \times 10^{32} \text{ eV}^{-1}$$

$$T_0 = 2.725 \text{ K} = 2.348 \times 10^{-4} \text{ eV}$$

$$T_{\nu 0} = (4/11)^{1/3} T_0$$

$$z_{\text{dec}} = 1100$$

$$m_e = 0.511 \text{ MeV}$$

$$m_N = 938 \text{ MeV}$$

$$m_p + m_e - m_H = 13.6 \text{ eV}$$

$$\zeta(3) = 1.20206$$

$$G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$$

$$M_{\text{Pl}} \equiv (8\pi G_N)^{-1/2} = 2.436 \times 10^{21} \text{ MeV}$$

$$g_n = g_p = g_e = 2, \quad g_H = 4$$

$$Q = m_n - m_p = 1.293 \text{ MeV}$$

$$g_*(T \ll m_e) = 3.363$$

$$g_{*S}(T \ll m_e) = 3.909$$

$$g_*(1 \text{ MeV}) = g_{*S}(1 \text{ MeV}) = 10.75$$

$$\tau_n = t_{1/2} / \ln 2 = 880 \text{ s}$$

$$n + \nu_e \leftrightarrow p + e^-$$

$$\mu_e \ll T \quad (\text{when } T > 30 \text{ keV})$$

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

$$n_i = \left\{ \begin{array}{l} 1 \\ 3/4 \end{array} \right\} \frac{\zeta(3)}{\pi^2} g_i T^3 \quad (T \gg m_i)$$

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-\frac{m_i - \mu_i}{T}} \quad (T \ll m_i)$$

$$3 \frac{\dot{a}^2}{a^2} + 3 \frac{K}{a^2} = 8\pi G_N \rho + \Lambda$$

$$3 \frac{\ddot{a}}{a} = -4\pi G_N (\rho + 3p) + \Lambda$$

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0$$

$$\rho = \frac{\pi^2}{30} g_* T^4$$

$$s = \frac{2\pi^2}{45} g_{*S} T^3$$

$$g_*^{1/2} t T^2 = 1.51 M_{\text{Pl}}$$

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$n - \bar{n} = \frac{gT^3}{6\pi^2} \left[\pi^2 \frac{\mu}{T} + \left(\frac{\mu}{T} \right)^3 \right] \quad (T \gg m)$$

$$n - \bar{n} = 2g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \sinh \frac{\mu}{T} \quad (T \ll m)$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \text{arsinh}(x)$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\varepsilon = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = M_{\text{Pl}}^2 \frac{V''}{V}$$

$$H^2 = \frac{V(\phi)}{3M_{\text{Pl}}^2}$$

$$3H\dot{\phi} = -V'(\phi)$$

$$\mathcal{R} = -H \frac{\delta\phi}{\dot{\phi}}$$

$$\mathcal{P}_g(k) \equiv \left(\frac{L}{2\pi} \right)^3 4\pi k^3 \langle |g_{\mathbf{k}}|^2 \rangle$$

$$\left(\frac{2\pi}{L} \right)^3 \sum_{\mathbf{k}} \rightarrow \int d^3k$$

$$\mathcal{P}_\varphi(k) = \left(\frac{H}{2\pi} \right)_{aH=k}^2$$

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}} \right)_{aH=k}^2 \left(\frac{H}{2\pi} \right)_{aH=k}^2$$

$$n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$$

$$\left(\frac{\delta T}{T} \right)_{\text{obs}} = \frac{1}{4} \delta_\gamma - \mathbf{v} \cdot \hat{\mathbf{n}} + \Phi(t_{\text{dec}}, \mathbf{x}_{\text{ls}}) + 2 \int \dot{\Phi} dt$$

$$e^{i\mathbf{k} \cdot \mathbf{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y_{lm}(\hat{\mathbf{x}}) Y_{lm}^*(\hat{\mathbf{k}})$$

$$\ddot{\Phi}_{\mathbf{k}} + H(4 + 3c_s^2) \dot{\Phi}_{\mathbf{k}} + c_s^2 \frac{k^2}{a^2} \Phi_{\mathbf{k}} + [2\dot{H} + (3 + 3c_s^2)H^2] \Phi_{\mathbf{k}} = 0$$

$$\delta_{\mathbf{k}} = -\frac{2}{3} \frac{k^2}{(aH)^2} \Phi_{\mathbf{k}} - 2 \frac{1}{H} \dot{\Phi}_{\mathbf{k}} - 2\Phi_{\mathbf{k}}$$

$$\Phi_{\mathbf{k}} = -\frac{3 + 3w}{5 + 3w} \mathcal{R}_{\mathbf{k}}, \quad (w \equiv p/\rho)$$

$$\int d\Omega Y_{lm}^*(\theta, \phi) Y_{l'm'}(\theta, \phi) = \delta_{ll'} \delta_{mm'}$$

$$\sum_m |Y_{lm}(\theta, \phi)|^2 = \frac{2l+1}{4\pi}$$

$$\int_0^\infty \frac{dz}{z} j_l(z)^2 = \frac{1}{2l(l+1)}$$

The particles in the Standard Model and $g_*(T)$

Particle Data Group, 2014

Quarks	t	173.2 ± 0.9 GeV	\bar{t}	spin $\frac{1}{2}$ 3 colours	$g = 2 \cdot 2 \cdot 3 = 12$	<hr/>
	b	4.18 ± 0.03 GeV	\bar{b}			
	c	1.275 ± 0.025 GeV	\bar{c}			
	s	95 ± 5 MeV	\bar{s}			
	d	$4.5\text{--}5.3$ MeV	\bar{d}			
	u	$1.8\text{--}3.0$ MeV	\bar{u}	72		
Gluons	8 massless bosons			spin 1	$g = 2$	16
Leptons	τ^-	1776.82 ± 0.16 MeV	τ^+	spin $\frac{1}{2}$	$g = 2 \cdot 2 = 4$	<hr/>
	μ^-	105.658 MeV	μ^+			
	e^-	510.999 keV	e^+			
	ν_τ	< 2 eV	$\bar{\nu}_\tau$	spin $\frac{1}{2}$	$g = 2$	<hr/>
	ν_μ	< 2 eV	$\bar{\nu}_\mu$			
	ν_e	< 2 eV	$\bar{\nu}_e$			
Electroweak gauge bosons	W^\pm	80.385 ± 0.015 GeV		spin 1	$g = 3$	<hr/>
	Z^0	91.1876 ± 0.0021 GeV				
	γ	0 ($< 1 \times 10^{-18}$ eV)				
Higgs boson	H^0	125.7 ± 0.4 GeV		spin 0	$g = 1$	1
						<hr/>
						$g_f = 72 + 12 + 6 = 90$
						$g_b = 16 + 11 + 1 = 28$

History of $g_*(T)$

$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW crossover	(no effect)	
$T < 170$ GeV	top annihilation	96.25	
$T < 120$ GeV	H^0	95.25	
$T < 80$ GeV	W^\pm, Z^0	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD crossover	17.25	(u,d,g \rightarrow $\pi^{\pm,0}$, $37 \rightarrow 3$)
$T < 100$ MeV	π^\pm, π^0, μ^-	10.75	$e^\pm, \nu, \bar{\nu}, \gamma$ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$