Cosmology I

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Preface

These are the lecture notes for my Cosmology course at the University of Helsinki. I first lectured Cosmology at the Helsinki University of Technology in 1996 and then at University of Helsinki from 1997 to 2009. Syksy Räsänen taught the course from 2010 to 2015. I have lectured the course again since 2016. These notes are based on my notes from 2009, but I have adopted some improvements made by Syksy. As the course progresses I will keep updating the lecture notes.

A difficulty in teaching cosmology is that some very central aspects of modern cosmology rely on rather advanced physics, like quantum field theory in curved spacetime. On the other hand, the main applications of these aspects can be discussed in relatively simple terms, so requiring students to have such background would seem overkill, and would prevent many interested students in getting a taste of this exciting and important subject. Thus I am assuming just the standard bachelor level theoretical physics background (mechanics, special relativity, quantum mechanics, statistical physics). The more advanced theories that cosmology relies on, general relativity and quantum field theory, are reviewed as a part of this course to a sufficient extent, that we can go on. This represents a compromise which requires from the student an acceptance of some results without a proper derivation. Even a quantum mechanics or statistical physics background is not necessary, if the student is willing to accept some results taken from these fields (in the beginning of Chapter 4). As mathematical background, Cosmology I requires integral and differential calculus (as taught in Matemaattiset apuneuvot I, II). Cosmology II requires also Fourier analysis and spherical harmonic analysis (Fysikan matemaattiset menetelmät I, II).

The course is divided into two parts. In Cosmology I, the universe is discussed in terms of the homogeneous and isotropic approximation (the Friedmann–Robertson–Walker model), which is good at the largest scales and in the early universe. In Cosmology II, deviations from this homogeneity and isotropy, i.e., the structure of the universe, are discussed. I thank Elina Keihänen, Jussi Väliviita, Ville Heikkilä, Reijo Keskitalo, and Elina Palmgren for preparing some of the figures and doing the calculations behind them.

– Hannu Kurki-Suonio, December 2017
1 Introduction

Cosmology is the study of the universe as a whole, its structure, its origin, and its evolution.

Cosmology is based on observations, mostly astronomical, and laws of physics. These lead naturally to the standard framework of modern cosmology, the Hot Big Bang.

As a science, cosmology has a severe restriction: there is only one universe. We cannot make experiments in cosmology, and observations are restricted to a single object: the Universe. Thus we can make no comparative or statistical studies among many universes. Moreover, we are restricted to observations made from a single location, our solar system. It is quite possible that due to this special nature of cosmology, some important questions can never be answered.

Nevertheless, the last few decades have seen a remarkable progress in cosmology, as a significant body of relevant observational data has become available with modern astronomical instruments. We now have a good understanding of the overall history and structure of the universe, but important open questions remain, e.g., the nature of dark matter and dark energy. Hopefully observations with more advanced instruments will resolve many of these questions in the coming decades.

The fundamental observation behind the big bang theory was the redshift of distant galaxies. Their spectra are shifted towards longer wavelengths. The further out they are, the larger is the shift. This implies that they are receding away from us; the distance between them and us is increasing. According to general relativity, we understand this as the expansion of the intergalactic space itself, not as actual motion of the galaxies. As the space expands, the wavelength of light traveling through space expands also.

This expansion appears to be uniform over large scales: the whole universe expands at the same rate. We describe this expansion by a time-dependent scale factor, $a(t)$. Starting from the observed present value of the expansion rate, $H \equiv (da/dt)/a \equiv \dot{a}/a$, and knowledge of the energy content of the universe, we can use general relativity to calculate $a(t)$ as a function of time. The result is, using the standard model of particle physics for the energy content at high energies, that $a(t) \to 0$ about 14 billion years ago (I use the American convention, adopted now also by the British, where billion $\equiv 10^9$). At this singularity, the “beginning” of the big bang, which we choose as the origin of our time coordinate, $t = 0$, the density of the universe $\rho \to \infty$. In reality, we do not expect the standard model of particle physics to be applicable at extremely high energy densities. Thus there should be modifications to this picture at the very earliest times, probably just within the first nanosecond. A popular modification, discussed in Cosmology II, is cosmological inflation, which extends these earliest times, possibly, like in the "eternal inflation" model, infinitely (although usually inflation is thought to last only a small fraction of a second). At the least, when the density becomes comparable to the so called Planck density, $\rho_{Pl} \sim 10^{96}$ kg/m$^3$, quantum gravitational effects should be large, so that general relativity itself

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1There may, in principle, exist other universes, but they are not accessible to our observation. We spell Universe with a capital letter when we refer specifically to the universe we live in, whereas we spell it without a capital letter, when we refer to the more general or theoretical concept of the universe. In Finnish, ‘maailmankaikkeus’ is not capitalized.

2Except for the very beginning.

3These are not the most fundamental viewpoints. In general relativity the universe is understood as a four-dimensional curved spacetime, and its separation into space and time is a coordinate choice, based on convenience. The concepts of expansion of space and photon wavelength are based on such a coordinate choice. The most fundamental aspect is the curvature of spacetime. At large scales, the spacetime is curved in such a way that it is convenient to view this curvature as expansion of space, and in the related coordinate system the photon wavelength is expanding at the corresponding rate.

4This applies only at distance scales larger than the scale of galaxy clusters, about 10 Mpc. Bound systems, e.g., atoms, chairs, you and me, the Earth, the solar system, galaxies, or clusters of galaxies, do not expand. The expansion is related to the overall averaged gravitational effect of all matter in the universe. Within bound systems local gravitational effects are much stronger, so this overall effect is not relevant.
is no longer valid. To describe this Planck era, we would need a theory of quantum gravity, which we do not have.\(^5\) Thus these earliest times, including \(t = 0\), have to be excluded from the scientific big bang theory. Nevertheless, when discussing the universe after the Planck era and/or after inflation we customarily set the origin of the time coordinate \(t = 0\), where the standard model solution would have the singularity.

Thus the proper way to understand the term “big bang”, is not as some event by which the universe started or came into existence, but as a period in the early universe, when the universe was very hot,\(^6\) very dense, and expanding rapidly.\(^7\) Moreover, the universe was then filled with an almost homogeneous “primordial soup” of particles, which was in thermal equilibrium for a long time. Therefore we can describe the state of the early universe with a small number of thermodynamic variables, which makes the time evolution of the universe calculable.

### 1.1 Misconceptions

There are some popular misconceptions about the big bang, which we correct here:

The universe did not start from a point. The part of the universe which we can observe today was indeed very small at very early times, possibly smaller than \(1\) mm in diameter at the earliest times that can be sensibly discussed within the big bang framework. And if the inflation scenario is correct, even very much smaller than that before (or during earlier parts of) inflation, so in that sense the word “point” may be appropriate. But the universe extends beyond what can be observed today (beyond our “horizon”), and if the universe is infinite (we do not know whether the universe is finite or infinite), in current models it has always been infinite, from the very beginning.

As the universe expands it is not expanding into some space “around” the universe. The universe contains all space, and this space itself is “growing larger”.\(^8\)

### 1.2 Units and terminology

We shall use natural units where \(c = \hbar = k_B = 1\).

#### 1.2.1 \(c = 1\)

Relativity theory unifies space and time into a single concept, the 4-dimensional spacetime. It is thus natural to use the same units for measuring distance and time. Since the (vacuum) speed of light is \(c = 299,792,458\) m/s, we set \(1\) s = \(299,792,458\) m, so that \(1\) second = \(1\) light second, \(1\) year = \(1\) light year, and \(c = 1\). Velocity is thus a dimensionless quantity, and smaller than one\(^9\) for massive objects. Energy and mass have now the same dimension, and Einstein’s famous equivalence relation between mass and energy, \(E = mc^2\), becomes \(E = m\). This justifies a change in terminology; since mass and energy are the same thing, we do not waste two words on it. As is customary in particle physics we shall use the word “energy”, \(E\), for the above

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\(^5\) *String theory* is a candidate for the theory of quantum gravity. It is, however, very difficult to calculate definite predictions for the very early universe from string theory. This is a very active research area at present, but remains quite speculative.

\(^6\) The realization that the early universe must have had a high temperature did not come immediately after the discovery of the expansion. The results of big bang nucleosynthesis and the discovery of the cosmic microwave background are convincing evidence that the Big Bang was Hot.

\(^7\) There is no universal agreement among cosmologists about what time period the term ”big bang” refers to. My convention is that it refers to the time from the end of inflation (or from whenever the standard hot big bang picture becomes valid) until recombination, so that it is actually a 380-000-year-long period, still short compared to the age of the universe.

\(^8\) If the universe is infinite, we can of course not apply this statement to the volume of the entire universe, which is infinite, but it applies to finite parts of the universe.

\(^9\) In the case of “physical” (as opposed to “coordinate”) velocities.
quantity. By the word “mass”, \( m \), we mean the rest mass. Thus we do not write \( E = m \), but \( E = m\gamma \), where \( \gamma = 1/\sqrt{1 - v^2} \). The difference between energy and mass, \( E - m \), is the kinetic energy of the object.\(^{10}\)

1.2.2 \( k_B = 1 \)

Temperature, \( T \), is a parameter describing a thermal equilibrium distribution. The formula for the equilibrium occupation number of energy level \( E \) includes the exponential form \( e^{\beta E} \), where the parameter \( \beta = 1/k_B T \). The only function of the Boltzmann constant, \( k_B = 1.3805 \times 10^{-23} \text{ J/K} \), is to convert temperature into energy units. Since we now decide to give temperatures directly in energy units, \( k_B \) becomes unnecessary. We define \( 1 \text{ K} = 1.3806 \times 10^{-23} \text{ J} \), or

\[
1 \text{ eV} = 11600 \text{ K} = 1.78 \times 10^{-36} \text{ kg} = 1.60 \times 10^{-19} \text{ J}. \tag{1}
\]

Thus \( k_B = 1 \), and the exponential form is just \( e^{E/T} \).

1.2.3 \( h = 1 \)

The third simplification in the natural system of units is to set the Planck constant \( h \equiv h/2\pi = 1 \). This makes the dimension of mass and energy 1/time or 1/distance. This time and distance give the typical time and distance scales quantum mechanics associates with the particle energy. For example, the energy of a photon \( E = h\omega = \omega = 2\pi\nu \) is equal to its angular frequency. We have

\[
1 \text{ eV} = 5.07 \times 10^6 \text{ m}^{-1} = 1.52 \times 10^{15} \text{ s}^{-1}. \tag{2}
\]

A useful relation to remember is

\[
h = 197 \text{ MeV fm} = 1, \tag{3}
\]

where we have the energy scale \( \sim 200 \text{ MeV} \) and length scale \( \sim 1 \text{ fm} \) of strong interactions. Equations become now simpler and the physical relations more transparent, since we do not have to include the above fundamental constants. This is not a completely free lunch, however; we often have to do conversions among the different units to give our answers in familiar units.

1.2.4 Astronomical units

A common unit of mass and energy is the solar mass, \( M_\odot = 1.99 \times 10^{30} \text{ kg} \), and a common unit of length is parsec, \( 1 \text{ pc} = 3.26 \text{ light years} = 3.09 \times 10^{16} \text{ m} \). One parsec is defined as the distance from which 1 astronomical unit (AU, the distance between the Earth and the Sun) forms an angle of one arcsecond, \( 1'' \). More common in cosmology is \( 1 \text{ Mpc} = 10^6 \text{ pc} \), which is a typical distance between neighboring galaxies. For angles, 1 degree (\( 1^\circ \)) = 60 arcminutes (60') = 3600 arcseconds (3600 '').

1.3 Brief History of the Early Universe

Because of the high temperature, particles had large energies in the early universe. To describe matter in that era, we need particle physics. The standard model of particle physics is called \( \text{SU}(3)_c \otimes \text{SU}(2)_w \otimes \text{U}(1)_y \), which describes the symmetries of the theory. From the viewpoint of the standard model, we live today in a low-energy universe, where many of the symmetries of the theory are broken. The “natural” energy scale of the theory is reached when the temperature of the universe exceeds 100 GeV (about \( 10^{15} \text{ K} \)), which was the case when the universe was younger than \( 10^{-11} \text{ s} \). Then the primordial soup of particles consisted of free massless fermions (quarks

\(^{10}\)The talk about “converting mass to energy” or vice versa can be understood to refer to conversion of rest mass into kinetic energy.
Figure 1: Short history of the universe.
and leptons) and massless gauge bosons mediating the interactions (color and electroweak) between these fermions. The standard model also includes a particle called the Higgs boson.

Higgs boson is responsible for the breaking of the electroweak (the SU(2) \( \times \) U(1)) symmetry. This is one of the phase transitions\(^{11}\) in the early universe. In the electroweak (EW) phase transition the electroweak interaction becomes two separate interactions: 1) the weak interaction mediated by the massive gauge bosons \( W^\pm \) and \( Z^0 \), and 2) the electromagnetic interaction mediated by the massless gauge boson \( \gamma \), the photon. Fermions acquire their masses in the EW phase transition.\(^{12}\) The mass is due to the interaction of the particle with the Higgs field.

The EW phase transition took place when the universe cooled below the critical temperature \( T_c \sim 100 \text{ GeV} \) of the phase transition at \( t \sim 10^{-11} \text{ s} \). See Fig. 1.

In addition to the standard model particles, the universe contains dark matter particles, whose exact nature is unknown. These will be discussed later, but we ignore them now for a while.

Another phase transition, the QCD (quantum chromodynamics) transition, or the quark–hadron transition, took place at \( t \sim 10^{-5} \text{ s} \). The critical temperature of the QCD phase transition is \( T_c \sim 150 \text{ MeV} \). Quarks, which had been free until this time, formed hadrons: baryons, e.g., the nucleons \( n \) and \( p \), and mesons, e.g., \( \pi \), \( K \). The matter filling the universe was converted from a quark–gluon plasma to a hadron gas.

To every type of particle there is a corresponding antiparticle, which has the same properties (e.g., mass and spin) as the particle, but its charges, like electric charge and color charge, have opposite sign. Particles which have no charges, like photons, are their own antiparticles. At high temperatures, \( T \gg m \), where \( m \) is the mass of the particle, particles and antiparticles are constantly created and annihilated in various reactions, and there is roughly the same number of particles and antiparticles. But when \( T \ll m \), particles and antiparticles may still annihilate each other (or decay, if they are unstable), but there is no more thermal production of particle–antiparticle pairs. As the universe cools, heavy particles and antiparticles therefore annihilate each other. These annihilation reactions produce additional lighter particles and antiparticles. If the universe had had an equal number of particles and antiparticles, only photons and neutrinos (of the known particles) would be left over today. The presence of matter today indicates that in the early universe there must have been slightly more nucleons and electrons than antinucleons and positrons, so that this excess was left over. The lightest known massive particle with strong or electroweak interactions is the electron,\(^{13}\) so the last annihilation event was the electron–positron annihilation which took place when \( T \sim m_e \sim 0.5 \text{ MeV} \) and \( t \sim 1 \text{ s} \). After this the only remaining antiparticles were the antineutrinos, and the primordial soup consisted of a large number of photons (who are their own antiparticles) and neutrinos (and antineutrinos) and a smaller number of “left-over” protons, neutrons, and electrons.

When the universe was a few minutes old, \( T \sim 100 \text{ keV} \), protons and neutrons formed nuclei of light elements. This event is known as Big Bang Nucleosynthesis (BBN), and it produced about 75% (of the total mass in ordinary matter) \(^1\text{H} \), 25% \(^4\text{He} \), \( 10^{-4} \) \(^2\text{H} \), \( 10^{-4} \) \(^3\text{He} \), and \( 10^{-9} \) \(^7\text{Li} \). (Other elements were formed much later, mainly in stars). At this time matter was completely ionized, all electrons were free. In this plasma the photons were constantly scattering from electrons, so that the mean free path of a photon between these scatterings was short. This means that the universe was opaque, not transparent to light.

The universe became transparent when it was 380 000 years old. At a temperature \( T \sim\)

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\(^{11}\)It may be that the EW and QCD phase transitions do not satisfy the technical definition of phase transition, but are instead just cross-overs, which means that they don’t have a sharp critical temperature, but rather correspond to a temperature interval. The exact nature of these transitions is an open research problem.

\(^{12}\)Except possibly neutrinos, the origin of whose masses in uncertain.

\(^{13}\)According to observational evidence from neutrino oscillations, neutrinos also have small masses. However, at temperatures less than the neutrino mass, the neutrino interactions are so weak that the neutrinos and antineutrinos cannot annihilate each other.
3000 K (∼ 0.25 eV), the electrons and nuclei formed neutral atoms, and the photon mean free path became longer than the radius of the observable universe. This event is called recombination (although it actually was the first combination of electrons with nuclei, not a recombination). Since recombination the primordial photons have been traveling through space mostly without scattering. We can observe them today as the cosmic microwave background (CMB). It is light from the early universe. We can thus “see” the big bang.

After recombination, the universe was filled with hydrogen and helium gas (with traces of lithium). The first stars formed from this gas when the universe was a few hundred million years old; but most of this gas was left as interstellar gas. The radiation from stars reionized the interstellar gas when the universe was 700 million years old.

1.4 Cosmological Principle

The ancients thought that the Earth is at the center of the Universe. This is an example of misconceptions that may result from having observations only from a single location (in this case, from the Earth). In the sixteenth century Nicolaus Copernicus proposed the heliocentric model of the universe, where Earth and the other planets orbited the Sun. This was the first step in moving “us” away from the center of the Universe. Later it was realized that neither the Sun, nor our galaxy, lies at the center of the Universe. This lesson has led to the Copernican principle: We do not occupy a privileged position in the universe. This is closely related to the Cosmological principle: The universe is homogeneous and isotropic.

Homogeneous means that all locations are equal, so that the universe appears the same no matter where you are. Isotropic means that all directions are equal, so that the universe appears the same no matter which direction you look at. Isotropy refers to isotropy with respect to some particular location, but 1) from isotropy with respect to one location and homogeneity follows isotropy with respect to every location, and 2) from isotropy with respect to all locations follows homogeneity.

There are two variants of the cosmological principle when applied to the real universe. As phrased above, it clearly does not apply at small scales: planets, stars, galaxies, and galaxy clusters are obvious inhomogeneities. In the first variant the principle is taken to mean that a homogeneous and isotropic model of the universe is a good approximation to the real universe at large scales (larger than the scale of galaxy clusters). In the second variant we add to this that the small-scale deviations from this model are statistically homogeneous and isotropic. This means that if we calculate the statistical properties of these inhomogeneities and anisotropies over a sufficiently large region, these statistical measures are the same for different such regions.

The Copernican principle is a philosophical viewpoint. Once you adopt it, observations lead to the first variant of the cosmological principle. CMB is highly isotropic and so is the distribution of distant galaxies, so we have solid observational support for isotropy with respect to our location. Direct evidence for homogeneity is weaker, but adopting the Copernican principle, we expect isotropy to hold also for other locations in the Universe, so that then the Universe should also be homogeneous. Thus we adopt the cosmological principle for the simplest model of the universe, which is an approximation to the true universe. This should be a good approximation at large scales, and in the early universe also for smaller scales.

The second variant of the cosmological principle cannot be deduced the same way from observations and the Copernican principle, but it follows naturally from the inflation scenario discussed in Cosmology II.

1.5 Structure Formation

CMB tells us that the early universe was very homogeneous, unlike the present universe, where matter has accumulated into stars and galaxies. The early universe had, however, very small
density variations, at the $10^{-5}$ to $10^{-3}$ level, which we see as small intensity variations of the CMB (the CMB anisotropy). Due to gravity, these slight overdensities have grown in time, and eventually became galaxies. This is called structure formation in the universe. The galaxies are not evenly distributed in space but form various structures, galaxy groups, clusters (large gravitationally bound groups), “filaments”, and “walls”, separated by large, relatively empty “voids”. This present large scale structure of the universe forms a significant body of observational data in cosmology, which we can explain fairly well by cosmological theory.

There are two parts to structure formation:

1. The origin of the primordial density fluctuations, the “seeds of galaxies”. These are believed to be due to some particle physics phenomenon in the very early universe, probably well before the EW transition. The particle physics theories applicable to this period are rather speculative. The currently favored explanation for the origin of primordial fluctuations is known as inflation. Inflation, discussed in Cosmology II, is not a specific theory, but it is a certain kind of behavior of the universe that could result from many different fundamental theories. Until the 1990s the main competitor to inflation was topological defects. Such defects (e.g., cosmic strings) may form in some phase transitions. The CMB data has ruled out topological defects at least as the main cause of structure formation.

2. The growth of these fluctuations as we approach the present time. The growth is due to gravity, but depends on the composition and total amount (average density) of matter and energy in the universe.

1.6 Dark Matter and Dark Energy

One of the main problems in cosmology today is that most of the matter and energy content of the universe appears to be in some unknown forms, called dark matter and dark energy. The dark matter problem dates back to 1930s, whereas the dark energy problem arose in late 1990s.

From the motions of galaxies we can deduce that the matter we can directly observe as stars and other “luminous matter” is just a small fraction of the total mass which affects the galaxy motions through gravity. The rest is dark matter, something which we observe only due to its gravitational effect. We do not know what most of this dark matter is. A smaller part of it is just ordinary, “baryonic”, matter, which consists of atoms (or ions and electrons) just like stars, but does not shine enough for us to notice it. Possibilities include planet-like bodies in interstellar space, “failed” stars (too small, $m < 0.07 M_\odot$, to ignite thermonuclear fusion) called brown dwarfs, old white dwarf stars, and tenuous intergalactic gas. In fact, in large clusters of galaxies the intergalactic gas\textsuperscript{14} is so hot that we can observe its radiation. Thus its mass can be estimated and it turns out to be several times larger than the total mass of the stars in the galaxies. We can infer that other parts of the universe, where this gas would be too cold to be observable from here, also contain significant amounts of thin gas; which thus is apparently the main component of baryonic dark matter (BDM). However, there is not nearly enough of it to explain the dark matter problem.

Beyond these mass estimates, there are more fundamental reasons (BBN, structure formation) why baryonic dark matter cannot be the main component of dark matter. Most of the dark matter must be non-baryonic, meaning that it is not made out of protons and neutrons\textsuperscript{15}. The only non-baryonic particles in the standard model of particle physics that could act as dark matter, are neutrinos. If neutrinos had a suitable mass, $\sim 1$ eV, the neutrinos left from the early universe would have a sufficient total mass to be a significant dark matter component.

\textsuperscript{14}This gas is ionized, so it should more properly be called plasma. Astronomers, however, often use the word “gas” also when it is ionized.

\textsuperscript{15}And electrons. Although technically electrons are not baryons (they are leptons), cosmologists refer to matter made out of protons, electrons, and neutrons as “baryonic”. The electrons are anyway so light, that most of the mass comes from the true baryons, protons and neutrons.
However, structure formation in the universe requires most of the dark matter to have different properties than neutrinos have. Technically, most of the dark matter must be “cold”, instead of “hot”. These are terms that just refer to the dynamics of the particles making up the matter, and do not further specify the nature of these particles. The difference between hot dark matter (HDM) and cold dark matter (CDM) is that HDM is made of particles whose velocities were large compared to escape velocities from the gravity of overdensities, when structure formation began, but CDM particles had small velocities. Neutrinos with $m \sim 1$ eV, would be HDM. An intermediate case is called warm dark matter (WDM). Structure formation requires that most of the dark matter is CDM, or possibly WDM, but the standard model of particle physics contains no suitable particles. Thus it appears that most of the matter in the universe is made out of some unknown particles.

Fortunately, particle physicists have independently come to the conclusion that the standard model is not the final word in particle physics, but needs to be “extended”. The proposed extensions to the standard model contain many suitable CDM particle candidates (e.g., neutralinos, axions). Their interactions with standard model particles would have to be rather weak to explain why they have not been detected so far. Since these extensions were not invented to explain dark matter, but were strongly motivated by particle physics reasons, the cosmological evidence for dark matter is good, rather than bad, news from a particle physics viewpoint.

In these days the term ”dark matter” usually refers to the nonbaryonic dark matter, and often excludes also neutrinos, so that it refers only to the unknown particles that are not part of the standard model of particle physics.

Since all the cosmological evidence for CDM comes from its gravitational effects, it has been suggested by some that it does not exist, and that these gravitational effects might instead be explained by suitably modifying the law of gravity at large distances. However, the suggested modifications do not appear very convincing, and the evidence is in favor of the CDM hypothesis. The gravitational effect of CDM has a role at many different levels in the history and structure of the universe, so it is difficult for a competing theory to explain all of them. Most cosmologists consider the existence of CDM as an established fact, and are just waiting for the eventual discovery of the CDM particle in the laboratory (perhaps produced with the Large Hadronic Collider (LHC) at CERN).

The situation with the so-called dark energy is different. While dark matter fits well into theoretical expectations, the status of dark energy is much more obscure. The accumulation of astronomical data relevant to cosmology has made it possible to determine the geometry and expansion history of the universe accurately. It looks like yet another component to the energy density of the universe is required to make everything fit, in particular to explain the observed acceleration of the expansion. This component is called “dark energy”. Unlike dark matter, which is clustered, the dark energy should be relatively uniform in the observable universe. And while dark matter has negligible pressure, dark energy should have large, but negative pressure. The simplest possibility for dark energy is a cosmological constant or vacuum energy. Unlike dark matter, dark energy was not anticipated by high-energy-physics theory, and it appears difficult to incorporate it in a natural way. Again, another possible explanation is a modification of the law of gravity at large distances. In the dark energy case, this possibility is still being seriously considered. The difference from dark matter is that there is more theoretical freedom, since there are fewer relevant observed facts to explain, and that the various proposed models for dark energy do not appear very natural. A nonzero vacuum energy by itself would be natural from quantum field theory considerations, but the observed energy scale is unnaturally low.
1 INTRODUCTION

1.7 Observable Universe

The observations relevant to cosmology are mainly astronomical. The speed of light is finite, and therefore, when we look far away, we also look back in time. The universe has been transparent since recombination, so more than 99.99% of the history of the universe is out there for us to see. (See Fig. 2.)

The most important channel of observation is the electromagnetic radiation (light, radio waves, X-rays, etc.) coming from space. We also observe particles, cosmic rays (protons, electrons, nuclei) and neutrinos coming from space. A new channel, opened in 2015 by the first observation by LIGO (Laser Interferometer Gravitational-wave Observatory), are gravitational waves from space. In addition, the composition of matter in the solar system has cosmological significance.

1.7.1 Big bang and the steady-state theory

In the 1950s observational data on cosmology was rather sparse. It consisted mainly of the redshifts of galaxies, which were understood to be due to the expansion of space. At that time there was still room for different basic theories of cosmology. The main competitors were the steady-state theory and the Big Bang theory.

The steady-state theory is also known as the theory of continuous creation, since it postulates that matter is constantly being created out of nothing, so that the average density of the universe stays the same despite the expansion. According to the steady-state theory the universe has always existed and will always exist and will always look essentially the same, so that there is no overall evolution.

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By 2017, it is already a disappointment that LHC has not yet found a dark matter particle.
According to the Big Bang theory, the universe had a beginning at a finite time ago in the past; the universe started at very high density, and as the universe expands its density goes down. In the Big Bang theory the universe evolves; it was different in the past, and it keeps changing in the future. The name “Big Bang” was given to this theory by Fred Hoyle, one of the advocates of the steady-state theory, to ridicule it. Hoyle preferred the steady-state theory on philosophical grounds; to him, an eternal universe with no evolution was preferable to an evolving one with a mysterious beginning.

Both theories treated the observed expansion of the universe according to Einstein’s theory of General Relativity. The steady-state theory added to it a continuous creation of matter, whereas the Big Bang theory “had all the creation in the beginning”.

The accumulation of further observational data led to the abandonment of the steady-state theory. These observations were: 1) the cosmic microwave background (predicted by the Big Bang theory, problematic for steady-state), 2) the evolution of cosmic radio sources (they were more powerful in the past, or there were more of them), and 3) the abundances of light elements and their isotopes (predicted correctly by the Big Bang theory).

By today the evidence has become so compelling that it appears extremely unlikely that the Big Bang theory could be wrong in any essential way, and the Big Bang theory has become the accepted basic framework, or “paradigm” of cosmology. Thus it has become arcane to talk about ”Big Bang theory“, when we are just referring to modern cosmology. The term ”Big Bang“ should be understood as originating from this historical context. Thus it refers to the present universe evolving from a completely different early stage: hot, dense, rapidly expanding and cooling, instead of being eternal and unchanging. There are still, of course, many open questions on the details, and the very beginning is still completely unknown.

1.7.2 Electromagnetic channel

Although the interstellar space is transparent (except for radio waves longer than 100 m, absorbed by interstellar ionized gas, and short-wavelength ultraviolet radiation, absorbed by neutral gas), Earth’s atmosphere is opaque except for two wavelength ranges, the optical window ($\lambda = 300–800$ nm), which includes visible light, and the radio window ($\lambda = 1$ mm–20 m). The atmosphere is partially transparent to infrared radiation, which is absorbed by water molecules in the air; high altitude and dry air favors infrared astronomy. Accordingly, the traditional branches of astronomy are optical astronomy and radio astronomy. Observations at other wavelengths have become possible only during the past few decades, from space (satellites) or at very high altitude in the atmosphere (planes, rockets, balloons).

From optical astronomy we know that there are stars in space. The stars are grouped into galaxies. There are different kinds of galaxies: 1) irregular, 2) elliptical, and 3) flat disks or spirals. Our own galaxy (the Galaxy, or Milky Way galaxy) is a disk. The plane of the disk can be seen (at a dark night) as a faint band – the milky way – across the sky.

Notable nearby galaxies are the Andromeda galaxy (M31) and the Magellanic clouds (LMC, Large Magellanic Cloud, and SMC, Small Magellanic Cloud). These are the only other galaxies that are visible to the naked eye. The Magellanic clouds (as well as the center of the Milky Way) lie too far south, however, to be seen from Finland. The number of galaxies that can be seen with powerful telescopes is many billions.

\[^{17}\]Thus the steady-state theory postulates a modification to known laws of physics, this continuous creation of matter out of nothing. The Big Bang theory, on the other hand, is based only on known laws of physics, but it leads to an evolution which, when extended backwards in time, leads eventually to extreme conditions where the known laws of physics can not be expected to hold any more. Whether there was “creation” or something else there, is beyond the realm of the Big Bang theory. Thus the Big Bang theory can be said to be “incomplete” in this sense, in contrast to the steady-state theory being complete in covering all of the history of the universe.
1 INTRODUCTION

1.7.3 Redshift and the Hubble law

Modern cosmology originated from the observation by Edwin Hubble\(^{19}\) (in about 1929) that the redshifts of galaxies were proportional to their distance. See Fig. 3. The light from distant galaxies is redder (has longer wavelength) when it arrives here. This redshift can be determined with high accuracy from the spectral lines of the galaxy. These lines are caused by transitions between different energy states of atoms, and thus their original wavelengths \(\lambda_0\) are known. The redshift \(z\) is defined as

\[
z = \frac{\lambda - \lambda_0}{\lambda_0} \quad \text{or} \quad 1 + z = \frac{\lambda}{\lambda_0}
\]

where \(\lambda\) is the observed wavelength. The redshift is observed to be independent of wavelength. The proportionality relation

\[
z = H_0 d
\]

is called the Hubble law, and the proportionality constant \(H_0\) the Hubble constant. Here \(d\) is the distance to the galaxy and \(z\) its redshift.

For small redshifts \((z \ll 1)\) the redshift can be interpreted as the Doppler effect due to the relative motion of the source and the observer. The distant galaxies are thus receding from us

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\(^{18}\)The expansion of the universe also contributes: the redshift makes distant stars fainter, and the different spacetime geometry also has an effect. Thus also the steady-state theory resolved Olbers’ paradox.

\(^{19}\)This proportionality was actually discovered by Lemaître before Hubble, but he published in a relatively unknown journal, so his discovery went unnoticed at the time.
with the velocity
\[ v = z. \] (6)

The further out they are, the faster they are receding. Astronomers often report the redshift in velocity units (i.e., km/s). Note that 1 km/s = 1/299792.458 = 0.000003356. Since the distances to galaxies are convenient to give in units of Mpc, the Hubble constant is customarily given in units of km/s/Mpc, although clearly its dimension is just 1/time or 1/distance.

This is, however, not the proper way to understand the redshift. The galaxies are not actually moving, but the distances between the galaxies are increasing because the intergalactic space between the galaxies is expanding, in the manner described by general relativity. We shall later derive the redshift from general relativity. It turns out that equations (5) and (6) hold only at the limit \( z \ll 1 \), and the general result, \( d(z) \), relating distance \( d \) and redshift \( z \) is more complicated (discussed in Chapter 3). In particular, the redshift increases much faster than distance for large \( z \), reaching infinity at finite \( d \). However, redshift is directly related to the expansion. The easiest way to understand the cosmological redshift is that the wavelength of traveling light expands with the universe. (We derive this result in Chapter 3.) Thus the universe has expanded by a factor \( 1 + z \) during the time light traveled from an object with redshift \( z \) to us.

While the redshift can be determined with high accuracy, it is difficult to determine the distance \( d \). See Fig. 3, right panel. The distance determinations are usually based on the cosmic distance ladder. This means a series of relative distance determinations between more nearby and faraway objects. The first step of the ladder is made of nearby stars, whose absolute distance can be determined from their parallax, their apparent motion on the sky due to our motion around the Sun. The other steps require “standard candles”, classes of objects with the same absolute luminosity (radiated power), so that their relative distances are inversely related to the square roots of their “brightness” or apparent luminosity (received flux density). Several steps are needed, since objects that can be found close by are too faint to be observed from very far away.

An important standard candle is a class of variable stars called Cepheids. They are so bright that they can be observed (with the Hubble Space Telescope) in other galaxies as far away as the Virgo cluster of galaxies, more than 10 Mpc away. There are many Cepheids in the LMC, and the distance to the LMC (about 50 kpc) is an important step in the distance ladder. For larger distances supernovae (a particular type of supernovae, called Type Ia) are used as standard candles.

Errors (inaccuracies) accumulate from step to step, so that cosmological distances, and thus the value of the Hubble constant, are not known accurately. This uncertainty of distance scale is reflected in many cosmological quantities. It is customary to give these quantities multiplied by the appropriate power of \( h \), defined by
\[ H_0 = h \cdot 100 \text{ km/s/Mpc}. \] (7)

Still in the 1980s different observers reported values ranging from 50 to 100 km/s/Mpc \((h = 0.5 \text{ to } 1)\).\(^{20}\)

It was a stated goal of the Hubble Space Telescope (HST) to determine the Hubble constant with 10% accuracy. As a result of some 10 years of observations the Hubble Space Telescope Key Project to Measure the Hubble Constant gave as their result in 2001 as [2]
\[ H_0 = 72 \pm 8 \text{ km/s/Mpc}. \] (8)

\(^{20}\)In fact, there were two “camps” of observers, one reporting values close to 50, the other close to 100, both claiming error estimates much smaller than the difference.
Modern observations have narrowed down the range and a recent value is \[ H_0 = 72.5 \pm 2.5 \text{ km/s/Mpc} \] 
\( (h = 0.725 \pm 0.025) \). Here the uncertainty \( (\pm 2.5) \) represents a 68\% confidence range, i.e., it is estimated 68\% probable that the true value lies in this range. (Unless otherwise noted, we give uncertainties as 68\% confidence ranges. If the probability distribution is the so-called normal (Gaussian) distribution, this corresponds to the standard deviation \( (\sigma) \) of the distribution, i.e., a 1\( \sigma \) error estimate.) Results from different observers are not all entirely consistent with this result, so that the contribution of systematic effects to the probable error may have been underestimated.\(^{21}\) To single-digit precision, we can use \( h = 0.7 \).

The largest observed redshifts of galaxies and quasars are about \( z \sim 9 \). Thus the universe has expanded by a factor of ten while the observed light has been on its way. When the light left such a galaxy, the age of the universe was only about 500 million years. At that time the first galaxies were just being formed. This upper limit in the observations is, however, not due to there being no earlier galaxies; such galaxies are just too faint due to both the large distance and the large redshift. There may well be galaxies with a redshift greater than 10. NASA is building a new space telescope, the James Webb Space Telescope\(^{22}\) (JWST), which would be able to observe these.

The Hubble constant is called a “constant”, since it is constant as a function of position. It is, however, a function of time, \( H(t) \), in the cosmological time scale. \( H(t) \) is called the Hubble parameter, and its present value is called the Hubble constant, \( H_0 \). In cosmology, it is customary to denote the present values of quantities with the subscript 0. Thus \( H_0 = H(t_0) \).

The galaxies are not exactly at rest in the expanding space. Each galaxy has its own peculiar motion \( v_{\text{gal}} \), caused by the gravity of nearby mass concentrations (other galaxies). Neighboring galaxies fall towards each other, orbit each other etc. Thus the redshift of an individual galaxy is the sum of the cosmic and the peculiar redshift.

\[
 z = H_0 d + \hat{n} \cdot v_{\text{gal}} \quad \text{(when } z \ll 1) .
\]

(Here \( \hat{n} \) is the “line-of-sight” unit vector giving the direction from the observer towards the galaxy.) Usually only the redshift is known precisely. Typically \( v_{\text{gal}} \) is of the order 500 km/s. (In large galaxy clusters, where galaxies orbit each other, it can be several thousand km/s; but then one can take the average redshift of the cluster.) For faraway galaxies, \( H_0 r \gg v_{\text{gal}} \), and the redshift can be used as a measure of distance. It is also related to the age of the universe at the observed time. Objects with a large \( z \) are seen in a younger universe (as the light takes a longer time to travel from this more distant object).

### Horizon

Because of the finite speed of light and the finite age of the universe, only a finite part of the universe is observable. Our horizon is at that distance from which light has just had time to reach us during the entire age of the universe. Were it not for the expansion of the universe, the distance to this horizon \( d_{\text{hor}} \) would equal the age of the universe, 14 billion light years (4300 Mpc). The expansion complicates the situation; we shall calculate the horizon distance later. For large distances the redshift grows faster than \((5)\). At the horizon \( z \to \infty \), i.e., \( d_{\text{hor}} = d(z = \infty) \). The universe has been transparent only for \( z < 1090 \) (after recombination), so the “practical horizon”, i.e., the limit to what we can see, lies already at \( z \sim 1090 \). The distances \( d(z = 1090) \)

\(^{21}\)We discuss in Chapter 9 how CMB observations\(^{[6]}\) lead to a, model-dependent, smaller value, \( H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc} \).

\(^{22}\)www.jwst.nasa.gov
Figure 4: The distribution of galaxies from the Sloan Digital Sky Survey (SDSS) and the horizon. We are at the center of this diagram. Each dot represents an observed galaxy. The empty sectors are regions not surveyed. The figure shows fewer galaxies further out, since only the brightest galaxies can be seen at large distances. The red color represents the primordial plasma through which we cannot see. This figure can be thought of as our past light cone seen from the “top” (compare to Fig. 2). We see the inner surface of this sphere as the cosmic microwave background (see Fig. 6). As time goes on the horizon recedes and we can see further out. The “Future comoving visibility limit” is how far one can eventually see in the very distant future, assuming the “Concordance Model” for the universe (Sec. 3.3). Because of the accelerated expansion of the universe it is not possible to reach the most distant galaxies we see (beyond the circle marked “Unreachable”), even if traveling at (arbitrarily close to) the speed of light. Fig. 5 zooms in to the center region marked with the dotted circle. Figure from Gott et al: “Map of the Universe” (2005) [1].
and \( d(z = \infty) \) are close to each other; \( z = 4 \) lies about halfway from here to horizon. The expansion of the universe complicates the concept of distance; the statements above refer to the comoving distance, defined later.

Thus the question of whether the universe is finite or infinite in space is somewhat meaningless. In any case we can only observe a finite region, enclosed in the sphere with radius \( d_{\text{hor}} \). Sometimes the word “universe” is used to denote just this observable part of the “whole” universe. Then we can say that the universe contains some \( 10^{11} \) or \( 10^{12} \) galaxies and about \( 10^{23} \) stars. Over cosmological time scales the horizon of course recedes and parts of the universe which are beyond our present horizon become observable. However, if the expansion keeps accelerating, as the observations indicate it has been doing already for several billion years, the observable region is already close to its maximum extent, and in the distant future galaxies which are now observable will disappear from our sight due to their increasing redshift.

### 1.7.5 Optical astronomy and the large scale structure

There is a large body of data relevant to cosmology from optical astronomy. Counting the number of stars and galaxies we can estimate the matter density they contribute to the universe. Counting the number density of galaxies as a function of their distance, we can try to determine whether the geometry of space deviates from Euclidean (as it might, according to general relativity). Evolution effects complicate the latter, and this approach never led to conclusive results.

From the different redshifts of galaxies within the same galaxy cluster we obtain their relative motions, which reflect the gravitating mass within the system. The mass estimates for galaxy clusters obtained this way are much larger than those obtained by counting the visible stars and galaxies in the cluster, pointing to the existence of dark matter.

From the spectral lines of stars and gas clouds we can determine the relative amounts of different elements and their isotopes in the universe.

The distribution of galaxies in space and their relative velocities tell us about the large scale structure of the universe. The galaxies are not distributed uniformly. There are galaxy groups and clusters. Our own galaxy belongs to a small group of galaxies called the Local Group. The Local Group consists of three large spiral galaxies: M31 (the Andromeda galaxy), M33 (the Triangulum galaxy\(^{23} \)), both M31 and M33 are named after the constellations they are located in), and the Milky Way, and about 30 smaller (dwarf) galaxies. The nearest large cluster is the Virgo Cluster. The grouping of galaxies into clusters is not as strong as the grouping of stars into galaxies. Rather the distribution of galaxies is just uneven; with denser and more sparse regions. The dense regions can be flat structures (“walls”) which enclose regions with a much lower galaxy density (“voids”). See Fig. 5. The densest concentrations are called galaxy clusters, but most galaxies are not part of any well defined cluster, where the galaxies orbit the center of the cluster.

### 1.7.6 Radio astronomy

The sky looks very different to radio astronomy. There are many strong radio sources very far away. These are galaxies which are optically barely observable. They are distributed isotropically, i.e., there are equal numbers of them in every direction, but there is a higher density of them far away (at \( z > 1 \)) than close by (\( z < 1 \)). The isotropy is evidence of the homogeneity of the universe at the largest scales – there is structure only at smaller scales. The dependence on distance is a time evolution effect. It shows that the universe is not static or stationary, but

\(^{23}\)Sometimes it is called the Pinwheel galaxy, but this name is also being used for M83, M99, and M101.
1 INTRODUCTION

Figure 5: The distribution of galaxies from SDSS. This figure shows observed galaxies that are within 2° of the equator and closer than 858 Mpc. The empty sectors are regions not surveyed. Figure from Gott et al: "Map of the Universe" (2005) [1].
evolves with time. Some galaxies are strong radio sources when they are young, but become weaker with age by a factor of more than 1000.

Cold gas clouds can be mapped using the 21 cm spectral line of hydrogen. The ground state \((n = 1)\) of hydrogen is split into two very close energy levels depending on whether the proton and electron spins are parallel or antiparallel (the hyperfine structure). The separation of these energy levels, the hyperfine structure constant, is 5.9 µeV, corresponding to a photon wavelength of 21 cm, i.e., radio waves. The redshift of this spectral line shows that redshift is independent of wavelength (the same for radio waves and visible light), as it should be according to standard theory.

1.7.7 Cosmic microwave background

At microwave frequencies the sky is dominated by the cosmic microwave background (CMB), which is highly isotropic, i.e., the microwave sky appears glowing uniformly without any features, unless our detectors are extremely sensitive to small contrasts. The electromagnetic spectrum of the CMB is the black body spectrum with a temperature of \(T_0 = 2.725 \pm 0.001 \text{ K}\) (COBE 1999 [4]). In fact, it follows the theoretical black body spectrum better than anything else we can observe or produce. There is no other plausible explanation for its origin than that it was produced in the Big Bang. It shows that the universe was homogeneous and in thermal
equilibrium at the time \((z = 1090)\) when this radiation originated. The redshift of the photons causes the temperature of the CMB to fall as \((1 + z)^{-1}\), so that its original temperature was about \(T = 3000 \text{ K}\).

The state of a system in thermal equilibrium is determined by just a small number of thermodynamic variables, in this case the temperature and density (or densities, when there are several conserved particle numbers). The observed temperature of the CMB and the observed density of the present universe allows us to fix the evolution of the temperature and the density of the universe, which then allows us to calculate the sequence of events during the Big Bang. That the early universe was hot and in thermal equilibrium is a central part of the Big Bang paradigm, and it is often called the Hot Big Bang to spell this out.

With sensitive instruments a small anisotropy can be observed in the microwave sky. This is dominated by the dipole anisotropy (one side of the sky is slightly hotter and the other side colder), with an amplitude of \(3362.1 \pm 1.0 \mu \text{K}\), or \(\Delta T/T_0 = 0.001234\). This is a Doppler effect due to the motion of the observer, i.e., the motion of the Solar System with respect to the radiating matter at our horizon. The velocity of this motion is \(v = \Delta T/T_0 = 369.8 \pm 0.1 \text{ km/s}\) and it is directed towards the constellation of Leo (galactic coordinates \(l = 264.02^\circ, b = 48.25^\circ\); equatorial coordinates RA \(11^h11^m46^s\), Dec \(-6^\circ57^\prime\)), near the autumnal equinox (where the ecliptic and the equator cross on the sky) \([5]\). It is due to two components, the motion of the Sun around the center of the Galaxy, and the peculiar motion of the Galaxy due to the gravitational pull of nearby galaxy clusters\(^{24}\).

When we subtract the effect of this motion from the observations (and look away from the plane of the Galaxy – the Milky Way also emits microwave radiation, but with a nonthermal spectrum) the true anisotropy of the CMB remains, with an amplitude of about \(3 \times 10^{-5}\), or 80 microkelvins.\(^{25}\) See Fig. 6. This anisotropy gives a picture of the small density variations in the early universe, the “seeds” of galaxies. Theories of structure formation have to match the small inhomogeneity of the order \(10^{-4}\) at \(z \sim 1090\) and the structure observed today \((z = 0)\).

1.8 Distance, luminosity, and magnitude

In astronomy, the radiated power \(L\) of an object, e.g., a star or a galaxy, is called its absolute luminosity. The flux density \(l\) (power per unit area) of its radiation here where we observe it, is called its apparent luminosity. Assuming Euclidean geometry, and that the object radiates isotropically, these are related as

\[
l = \frac{L}{4\pi d^2},
\]

where \(d\) is our distance to the object. For example, the Sun has

\[
L_\odot = 3.9 \times 10^{26} \text{ W} \quad d_\odot = 1.496 \times 10^{11} \text{ m} \quad l_\odot = 1370 \text{ W/m}^2.
\]

\(^{24}\)Sometimes it is asked whether there is a contradiction with special relativity here – doesn’t CMB provide an absolute reference frame? There is no contradiction. The relativity principle just says that the laws of physics are the same in the different reference frames. It does not say that systems cannot have reference frames which are particularly natural for that system, e.g., the center-of-mass frame or the laboratory frame. For road transportation, the surface of the Earth is a natural reference frame. In cosmology, CMB gives us a good “natural” reference frame – it is closely related to the center-of-mass frame of the observable part of the universe, or rather, a part of it which is close to the horizon (the last scattering surface). There is nothing absolute here; the different parts of the plasma from which the CMB originates are moving with different velocities (part of the \(3 \times 10^{-5}\) anisotropy is due to these velocity variations); we just take the average of what we see. If there is something surprising here, it is that these relative velocities are so small, of the order of just a few km/s; reflecting the astonishing homogeneity of the early universe over large scales. We shall return to the question, whether these are natural initial conditions, later, when we discuss inflation.

\(^{25}\)The numbers refer to the standard deviation of the CMB temperature on the sky. The hottest and coldest spots deviate some 4 or 5 times this amount from the average temperature.
The ancients classified the stars visible to the naked eye into six classes according to their brightness. The concept of magnitude in modern astronomy is defined so that it roughly matches this ancient classification, but it is a real number, not an integer. The magnitude scale is a logarithmic scale, so that a difference of 5 magnitudes corresponds to a factor of 100 in luminosity. Thus a difference of 1 magnitude corresponds to a factor \(100^{1/5} = 2.512\). The absolute magnitude \(M\) and the apparent magnitude \(m\) of an object are defined as

\[
M \equiv -2.5 \log \frac{L}{L_0}
\]

\[
m \equiv -2.5 \log \frac{l}{l_0},
\]

(12)

where \(L_0\) and \(l_0\) are reference luminosities (\(\log \equiv \log_{10}\)). There are actually different magnitude scales corresponding to different regions of the electromagnetic spectrum, with different reference luminosities. The bolometric magnitude and luminosity refer to the power or flux integrated over all frequencies, whereas the visual magnitude and luminosity refer only to the visible light. In the bolometric magnitude scale \(L_0 = 3.0 \times 10^{28}\) W. The reference luminosity \(l_0\) for the apparent scale is chosen so in relation to the absolute scale that a star whose distance is \(d = 10\) pc has \(m = M\) (exercise: find the value of \(l_0\)). From this, (11), and (12) follows that the difference between the apparent and absolute magnitudes are related to distance as

\[
m - M = -5 + 5 \log d(\text{pc})
\]

(13)

This difference is called the distance modulus, and often astronomers just quote the distance modulus, when they have determined the distance to an object. If two objects are known to have the same absolute magnitude, but the apparent magnitudes differ by 5, we can conclude that the fainter one is 10 times farther away (assuming Euclidean geometry).

For the Sun we have

\[
M = 4.79 \quad (\text{visual})
\]

\[
M = 4.72 \quad (\text{bolometric})
\]

and

\[
m = -26.78 \quad (\text{visual}),
\]

(14)

where the apparent magnitude is as seen from Earth. Note that the smaller the magnitude, the brighter the object.

**Exercises**

The first three exercises are not based on these lecture notes. They should be doable with your previous physics background.

**Nuclear cosmochronometers.** The uranium isotopes 235 and 238 have half-lives \(t_{1/2}(235) = 0.704 \times 10^9\) a \(\text{ja } t_{1/2}(238) = 4.47 \times 10^9\) a. The ratio of their abundances on Earth is \(\frac{235\text{U}}{238\text{U}} = 0.00725\). When were they equal in abundance? The heavy elements were created in supernova explosions and mixed with the interstellar gas and dust, from which the earth was formed. According to supernova calculations the uranium isotopes are produced in ratio \(\frac{235\text{U}}{238\text{U}} = 1.3 \pm 0.2\). What does this tell us about the age of the Earth and the age of the Universe?

**Olbers’ paradox.**

1. Assume the universe is infinite, eternal, and unchanging (and has Euclidean geometry). For simplicity, assume also that all stars are the same size as the sun, and distributed evenly in space. Show that the line of sight meets the surface of a star in every direction, sooner or later. Use Euclidean geometry.
2. Let’s put in some numbers: The luminosity density of the universe is $10^8 L_\odot$/Mpc$^3$ (within a factor of 2). With the above assumption we have then a number density of stars $n_* = 10^8$ Mpc$^{-3}$. The radius of the sun is $r_\odot = 7 \times 10^8$ m. Define $r_{1/2}$ so that stars closer than $r_{1/2}$ cover 50% of the sky. Calculate $r_{1/2}$.

3. Let’s assume instead that stars have finite ages: they all appeared $t_\odot = 4.6 \times 10^9$ a ago. What fraction $f$ of the sky do they cover? What is the energy density of starlight in the universe, in kg/m$^3$? (The luminosity, or radiated power, of the sun is $L_\odot = 3.85 \times 10^{26}$ W).

4. Calculate $r_{1/2}$ and $f$ for galaxies, using $n_G = 3 \times 10^{-3}$ Mpc$^{-3}$, $r_G = 10$ kpc, and $t_G = 10^{10}$ a.

**Newtonian cosmology.** Use Euclidean geometry and Newtonian gravity, so that we interpret the expansion of the universe as an actual motion of galaxies instead of an expansion of space itself. Consider thus a spherical group of galaxies in otherwise empty space. At a sufficiently large scale you can treat this as a homogeneous cloud (the galaxies are the cloud particles). Let the mass density of the cloud be $\rho(t)$. Assume that each galaxy moves according to Hubble’s law $v(t,r) = H(t)r$. The expansion of the cloud slows down due to its own gravity. What is the acceleration as a function of $\rho$ and $r \equiv |r|$? Express this as an equation for $H(t)$ (here the overdot denotes time derivative). Choose some reference time $t = t_0$ and define $a(t) \equiv r(t)/r(t_0)$. Show that $a(t)$ is the same function for each galaxy, regardless of the value of $r(t_0)$. Note that $\dot{a}(t) = a(t_0)a(t)^{-3}$. Rewrite your differential equation for $H(t)$ as a differential equation for $a(t)$. You can solve $H(t)$ also using energy conservation. Denote the total energy (kinetic + potential) of a galaxy per unit mass by $\kappa$. Show that $K = -2\kappa/r(t_{0})^{2}$ has the same value for each galaxy, regardless of the value of $r(t_{0})$. Relate $H(t)$ to $\rho(t_{0})$, $K$, and $a(t)$. Whether the expansion continues forever, or stops and turns into a collapse, depends on how large $H$ is in relation to $\rho$. Find out the critical value for $H$ (corresponding to the escape velocity for the galaxies) separating these two possibilities. Turn the relation around to give the critical density corresponding to a given “Hubble constant” $H$. What is this critical density (in kg/m$^3$) for $H = 70$ km/s/Mpc?

**Practice with natural units.**

1. The Planck mass is defined as $M_{Pl} \equiv \frac{1}{\sqrt{8\pi G}}$, where $G$ is Newton’s gravitational constant. Give Planck mass in units of kg, J, eV, K, m$^{-1}$, and s$^{-1}$.

2. The energy density of the cosmic microwave background is $\rho_c = \frac{2}{15} T^4$ and its photon density is $n_c = \frac{1}{4\pi} \zeta(3) T^3$, where $\zeta$ is Riemann’s zeta function and $\zeta(3) = 1.20206$. What is this energy density in units of kg/m$^3$ and the photon density in units of m$^{-3}$, i) today, when $T = 2.725$ K, ii) when the temperature was $T = 1$ MeV? What was the average photon energy, and what was the wavelength and frequency of such an average photon?

3. Suppose the mass of an average galaxy is $m_G = 10^{11} M_\odot$ and the galaxy density in the universe is $n_G = 3 \times 10^{-3}$ Mpc$^{-3}$. What is the galactic contribution to the average mass density of the universe, in kg/m$^3$?

4. The critical density for the universe is $\rho_{crit} \equiv \frac{3}{8 \pi G} H_0^2$, where $H_0$ is the Hubble constant, whose value we take to be 70 km/s/Mpc. How much is the critical density in units of kg/m$^3$ and in MeV$^2$? What fraction of the critical density is contributed by the microwave background (today), by starlight (see earlier exercise above), and by galaxies?

**Reference luminosity.** Find the value of $l_0$ for the bolometric scale.

References


