5 Big Bang Nucleosynthesis

One quarter (by mass) of the baryonic matter in the universe is helium. Heavier elements make up a few per cent. The rest, i.e., the major part, is hydrogen.

The building blocks of atomic nuclei, the nucleons, or protons and neutrons, formed in the QCD transition at $T \sim 150 \text{ MeV}$ and $t \sim 20 \mu \text{s}$. The protons are hydrogen ($^1\text{H}$) nuclei.

Elements (their nuclei) heavier than helium, and also some of the helium, have mostly been produced by stars in different processes (see Fig. 1). However, the amount of helium and the presence of significant amounts of the heavier hydrogen isotope, deuterium ($^2\text{H}$), in the universe cannot be understood by these astrophysical mechanisms. It turns out that $^2\text{H}$, $^3\text{He}$, $^4\text{He}$, and a significant part of $^7\text{Li}$, were mainly produced already in the big bang, in a process we call Big Bang Nucleosynthesis (BBN).

The nucleons and antinucleons annihilated each other soon after the QCD transition, and the small excess of nucleons left over from annihilation did not have a significant effect on the expansion and thermodynamics of the universe until much later ($t \sim t_{\text{eq}} = \Omega^{-2}m_{\text{h}}^{-4}1000 \text{ a}$), when the universe became matter-dominated. The ordinary matter in the present universe comes from this small excess of nucleons. Let us now consider what happened to it in the early universe. We shall focus on the period when the temperature fell from $T \sim 10 \text{ MeV}$ to $T \sim 10 \text{ keV}$ ($t \sim 10 \text{ ms} - \text{few h}$).

5.1 Equilibrium

The total number of nucleons stays constant due to baryon number conservation. This baryon number can be in the form of protons and neutrons or atomic nuclei. Weak nuclear reactions may convert neutrons and protons into each other and strong nuclear reactions may build nuclei from them.

During the period of interest the nucleons and nuclei are nonrelativistic ($T \ll m_p$). Assuming thermal equilibrium we have

$$n_i = g_i \left( \frac{m_i T}{2 \pi} \right)^{3/2} e^{\mu_i/m_i}$$

for the number density of nucleus type $i$. If the nuclear reactions needed to build nucleus $i$ (with mass number $A$ and charge $Z$) from the nucleons,

$$(A - Z)n + Zp \leftrightarrow i,$$

occur at sufficiently high rate to maintain chemical equilibrium, we have

$$\mu_i = (A - Z)\mu_n + Z\mu_p$$

for the chemical potentials. Since for free nucleons

$$n_p = 2 \left( \frac{m_p T}{2 \pi} \right)^{3/2} e^{\mu_p/m_p}$$

$$n_n = 2 \left( \frac{m_n T}{2 \pi} \right)^{3/2} e^{\mu_n/m_n},$$

we can express $n_i$ in terms of the neutron and proton densities,

$$n_i = g_i A^{3/2} 2^{-A} \left( \frac{2 \pi}{m_N T} \right)^{3/2} n_p^{Z} n_n^{A-Z} e^{B_i/T},$$

where

$$B_i \equiv Zm_p + (A - Z)m_n - m_i$$
Figure 1: Astronomy Picture of the Day, 2017 October 24 (https://apod.nasa.gov/apod/ap171024.html: "Explanation: The hydrogen in your body, present in every molecule of water, came from the Big Bang. There are no other appreciable sources of hydrogen in the universe. The carbon in your body was made by nuclear fusion in the interior of stars, as was the oxygen. Much of the iron in your body was made during supernovas of stars that occurred long ago and far away. The gold in your jewelry was likely made from neutron stars during collisions that may have been visible as short-duration gamma-ray bursts or gravitational wave events. Elements like phosphorus and copper are present in our bodies in only small amounts but are essential to the functioning of all known life. The featured periodic table is color coded to indicate humanity’s best guess as to the nuclear origin of all known elements. The sites of nuclear creation of some elements, such as copper, are not really well known and are continuing topics of observational and computational research.")

In more scientific terms: During most of their lifetime (the main sequence phase), stars are powered by nuclear fusion of hydrogen into helium in their cores. When hydrogen is exhausted in the core they begin to fuse helium into heavier elements (the giant phase). How far this proceeds depends on the mass of the star. In the heaviest stars fusion proceeds all the way to iron ($^{56}\text{Fe}$). Beyond iron, fusion will no longer produce energy, since $^{56}\text{Fe}$ maximizes binding energy per nucleon. Heavier elements are thus produced in processes which need an energy source to power them. Some of the energy released by nuclear fusion in the stellar cores goes into production of these heavier elements in the giant phase. When the fusion energy is exhausted the star “dies”: lighter stars collapse into white dwarfs, heavier stars explode—this explosion is called a supernova. A supernova begins with a collapse as the pressure produced by the fusion longer supports the outer parts. This brings in and raises the temperature of unburnt nuclear fuel from the outer parts. The fusion of this material and the gravitational energy from the collapse release a lot of energy in a short time causing the explosion, which is one source of heavier elements. Also white dwarfs may become supernovae later, if they accrete more mass from companion stars. In all these dying/exploding cases, the outer parts of the stars are ejected and mix into the interstellar material. In a supernova explosion of a massive star the inner part collapses into a neutron star. Collisions of these neutron stars are another source of heavy elements. The main type of nuclear reaction responsible for the production of the heavier elements beyond iron is neutron capture. Since neutrons are neutral it is easy for them to penetrate a nucleus and raise the mass number of the nucleus. The resulting new nucleus may be unstable so that it will β decay, i.e., the neutron releases an electron (and an antineutrino) and becomes a proton. In a slow neutron capture process (s-process) this decay happens before another neutron is captured, and in a rapid neutron capture process (r-process) many neutrons are captured before such decay. Heavy elements are produced by the s-process in the giant phase where fusion reactions provide the required energetic neutrons. The r-process requires a high density of neutrons. It is thought that the main site for the r-process is provided by collisions of neutron stars. Beryllium and boron are mainly produced by cosmic rays breaking up heavier nuclei in interstellar space (cosmic ray spallation).
is the binding energy of the nucleus. Here we have approximated $m_p \approx m_n \approx m_i/A$ outside the exponent, and denoted it by $m_N$ ("nucleon mass").

<table>
<thead>
<tr>
<th>$^A_Z$</th>
<th>$B$(MeV)</th>
<th>$B/A$(MeV)</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^2$H</td>
<td>2.2245</td>
<td>1.11</td>
<td>3</td>
</tr>
<tr>
<td>$^3$H</td>
<td>8.4820</td>
<td>2.83</td>
<td>2</td>
</tr>
<tr>
<td>$^3$He</td>
<td>7.7186</td>
<td>2.57</td>
<td>2</td>
</tr>
<tr>
<td>$^4$He</td>
<td>28.2970</td>
<td>7.07</td>
<td>1</td>
</tr>
<tr>
<td>$^6$Li</td>
<td>31.9965</td>
<td>5.33</td>
<td>3</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>39.2460</td>
<td>5.61</td>
<td>4</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>37.6026</td>
<td>5.37</td>
<td>1</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>92.1631</td>
<td>7.68</td>
<td>1</td>
</tr>
<tr>
<td>$^{56}$Fe</td>
<td>492.2623</td>
<td>8.79</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. Some of the lightest nuclei (+ iron) and their binding energies.

The different number densities add up to the total baryon number density

$$\sum A_i n_i = n_B.$$  \hspace{2cm} (6)

The baryon number density $n_B$ can be expressed in terms of photon density

$$n_\gamma = \frac{2}{\pi^2} \zeta(3) T^3$$  \hspace{2cm} (7)

and the baryon/photon -ratio

$$\frac{n_B}{n_\gamma} = \frac{g_{ss}(T)}{g_{ss}(T_0)} \eta$$  \hspace{2cm} (8)

as

$$n_B = \frac{g_{ss}(T)}{g_{ss}(T_0)} \frac{2}{\pi^2} \zeta(3) T^3.$$  \hspace{2cm} (9)

After electron-positron annihilation $g_{ss}(T) = g_{ss}(T_0)$ and $n_B = \eta n_\gamma$. Here $\eta$ is the present baryon/photon ratio. It can be estimated from various observations in a number of ways. It’s order of magnitude is $10^{-9}$.

We define the mass fraction of nucleus $i$ as

$$X_i = \frac{A_i n_i}{n_B} \quad \text{so that} \quad \sum X_i = 1.$$  \hspace{2cm} (10)

The equilibrium mass fractions are, from Eq. (4),

$$X_i = \frac{1}{2} X_p^Z X_n^{A-Z} g_i A^2 \epsilon A^{-1} e^{B_i/T}$$  \hspace{2cm} (11)

where

$$\epsilon = \frac{1}{2} \left( \frac{2\pi}{m_N T} \right)^{3/2} n_B = \frac{1}{\pi^2} \zeta(3) \left( \frac{2\pi T}{m_N} \right)^{3/2} \frac{g_{ss}(T)}{g_{ss}(T_0)} \eta \sim \left( \frac{T}{m_N} \right)^{3/2} \eta.$$

The factors which change rapidly with $T$ are $\epsilon A^{-1} e^{B_i/T}$. For temperatures $m_N \gg T \gtrsim B_i$ we have $e^{B_i/T} \sim 1$ and $\epsilon \ll 1$. Thus $X_i \ll 1$ for others ($A > 1$) than protons and neutrons. As temperature falls, $\epsilon$ becomes even smaller and at $T \sim B_i$ we have $X_i \ll 1$ still. The temperature has to fall below $B_i$ by a large factor before the factor $e^{B_i/T}$ wins and the equilibrium abundance becomes large. We calculate below that, e.g., for $^4$He this happens at $T \sim 0.3$ MeV, and for $^2$H at $T \sim 0.07$ MeV. Thus we have initially only free neutrons and protons in large numbers.
5.2 Neutron-proton ratio

What can we say about $n_p$ and $n_n$? Protons and neutrons are converted into each other in the weak reactions

\[
\begin{align*}
n + \nu_e & \leftrightarrow p + e^- \\
n + e^+ & \leftrightarrow p + \bar{\nu}_e \\
n & \leftrightarrow p + e^- + \bar{\nu}_e.
\end{align*}
\]

If these reactions are in equilibrium, $\mu_n + \mu_{\nu_e} = \mu_p + \mu_e$, and the neutron/proton ratio is

\[
\frac{n_n}{n_p} = e^{-Q/T + (\mu_e - \mu_{\nu_e})/T},
\]

where $Q \equiv m_n - m_p = 1.293 \text{ MeV}$.

We need now some estimate for the chemical potentials of electrons and electron neutrinos. The universe is electrically neutral\(^1\), so the net number of electrons ($e^- - n_{e^+}$) equals the number of protons, and $\mu_e$ can be calculated exactly in terms of $\eta$ and $T$. We leave the exact calculation as an exercise, but give below a rough estimate for the ultrarelativistic limit ($T > m_e$):

In the ultrarelativistic limit

\[
n_{e^-} - n_{e^+} = \frac{2T^3}{6\pi^2} \left( \frac{\mu_e}{T} \right) + \left( \frac{\mu_e}{T} \right)^3 = n_\nu^* \approx n_B \approx \eta m_\gamma = \frac{2}{\pi^2} \zeta(3) T^3.
\]

Here $n_\nu^*$ includes the protons inside nuclei. Since $\eta$ is small, we must have $\mu_e \ll T$, and we can drop the $(\mu_e/T)^3$ term to get

\[
\frac{\mu_e}{T} \approx \frac{6}{\pi^2} \zeta(3) \eta.
\]

Thus $\mu_e/T \sim \eta \sim 10^{-9}$. The nonrelativistic limit can be done in a similar manner (exercise). It turns out that $\mu_e$ rises as $T$ falls, and somewhere between $T = 30 \text{ keV}$ and $T = 10 \text{ keV} \mu_e$ becomes larger than $T$, and, in fact, comparable to $m_e$.

For $T \gtrsim 30 \text{ keV}$, $\mu_e \ll T$, and we can drop the $\mu_e$ in Eq. (13).

Since we cannot detect the cosmic neutrino background, we don’t know the neutrino chemical potentials. Usually it is assumed that also all three $\mu_{\nu_i} \ll T$, so that the difference in the number of neutrinos and antineutrinos is small. Thus we ignore both $\mu_e$ and $\mu_{\nu_e}$, so that $\mu_p = \mu_n$ and the equilibrium neutron/proton ratio is

\[
\frac{n_n}{n_p} = e^{-Q/T}.
\]

(This is not valid for $T \lesssim 30 \text{ keV}$, since $\mu_e$ is no longer small, but we shall use this formula only at higher temperatures as will be seen below.)

For $T > 0.3 \text{ MeV}$, we still have $X_n + X_p \approx 1$, so the equilibrium abundances are

\[
X_n = \frac{e^{-Q/T}}{1 + e^{-Q/T}} \quad \text{and} \quad X_p = \frac{1}{1 + e^{-Q/T}}.
\]

Nucleons follow these equilibrium abundances until neutrinos decouple at $T \sim 1 \text{ MeV}$, shutting off the weak $n \leftrightarrow p$ reactions. After this the neutrons decay into protons, so that

\[
X_n(t) = X_n(t_d) e^{-(t-t_d)/\tau_n},
\]

where $\tau_n = 880.2 \pm 1.0 \text{ s}$ is the mean lifetime of a free neutron\(^2\). (The half-life is $\tau_{1/2} = (\ln 2) \tau_n$.)

\(^1\)Electromagnetism is stronger than gravity by a factor of about $10^{38}$ so that the possible excess in positive or negative charge should be much less than one per this number or otherwise it would have been noticed.

\(^2\)This value given for $\tau_n$ by the Particle Data Group has quite recently changed by much more than the claimed accuracy. From 2006 to 2010 the given value was $885.7 \pm 0.8 \text{ s}$.
5.3 Bottlenecks

Using (4), (16), and (6)\(^3\), we get all equilibrium abundances as a function of \(T\) (they also depend on the value of the parameter \(\eta\)). There are two items to note, however:

1. The normalization condition, Eq. (6), includes all nuclei up to uranium etc. Thus we would get a huge polynomial equation from which to solve \(X_p\). (After one has \(X_p\), one gets the rest easily from (4) and (16).)

2. In practice we don’t have to care about the first item, since as the temperature falls the nuclei no longer follow their equilibrium abundances. The reactions are in equilibrium only at high temperatures, when the other equilibrium abundances except \(X_p\) and \(X_n\) are small, and we can use the approximation \(X_n + X_p = 1\).

In the early universe the baryon density is too low and the time available is too short for reactions involving three or more incoming nuclei to occur at any appreciable rate. The heavier nuclei have to be built sequentially from lighter nuclei in two-particle reactions, so that deuterium is formed first in the reaction

\[
\text{n} + \text{p} \rightarrow \text{d} + \gamma.
\]

Only when deuterons are available can helium nuclei be formed, and so on. This process has “bottlenecks”: the lack of sufficient densities of lighter nuclei hinders the production of heavier nuclei, and prevents them from following their equilibrium abundances.

As the temperature falls, the equilibrium abundances rise fast. They become large later for nuclei with small binding energies. Since deuterium is formed directly from neutrons and protons it can follow its equilibrium abundance as long as there are large numbers of free neutrons available. Since the deuterium binding energy is rather small, the deuterium abundance becomes large rather late (at \(T < 100\text{ keV}\)). Therefore heavier nuclei with larger binding energies, whose equilibrium abundances would become large earlier, cannot be formed. This is the deuterium bottleneck. Only when there is lots of deuterium (\(X_d \sim 10^{-3}\)), can helium be produced in large numbers.

The nuclei are positively charged and there is thus an electromagnetic repulsion between them. The nuclei need thus large kinetic energies to overcome this Coulomb barrier and get within the range of the strong interaction. Thus the cross sections for these fusion reactions fall rapidly with energy and the nuclear reactions are “shut off” when the temperature falls below \(T \sim 30\text{ keV}\). Thus there is less than one hour available for nucleosynthesis. Because of additional bottlenecks (e.g., there are no stable nuclei with \(A = 8\)) and the short time available, only very small amounts of elements heavier than helium are formed.

5.4 Calculation of the helium abundance

Let us now calculate the numbers. We saw that because of the deuterium bottleneck, \(X_n + X_p \approx 1\) holds until \(T \sim 0.1\text{ MeV}\). Until then, we get \(X_n\) and \(X_p\) at first from (17) and then from (18). In reality, neutrino decoupling and thus the shift from behavior (17) to behavior (18) is not instantaneous, but an approximation where one takes it to be instantaneous at time \(t_1\) when \(T = 0.8\text{ MeV}\), so that \(X_n(t_1) = 0.1657\), gives a fairly accurate final result.

Deuterium has \(B_d = 2.22\text{ MeV}\), and we get \(ee^{B_d/T} = 1\) at \(T_d = 0.06\text{ MeV} - 0.07\text{ MeV}\) (assuming \(\eta = 10^{-10} - 10^{-8}\)), so the deuterium abundance becomes large near this temperature. Since \(^4\text{He}\) has a much higher binding energy, \(B_4 = 28.3\text{ MeV}\), the corresponding situation \(ee^{B_4/T} = 1\) occurs at a higher temperature \(T_4 \sim 0.3\text{ MeV}\). But we noted earlier that only deuterium stays

\(^3\)For \(n_p\) and \(n_n\) we know just their ratio, since we do not know \(\mu_p\) and \(\mu_n\), only that \(\mu_p = \mu_n\). Therefore this extra equation is needed to solve all \(n_i\).
close to its equilibrium abundance once it gets large. Helium begins to form only when there is sufficient deuterium available, in practice slightly above $T_d$. Helium forms then rapidly. The available number of neutrons sets an upper limit to $^4$He production. Since helium has the highest binding energy per nucleon (of all isotopes below $A=12$), almost all neutrons end up in $^4$He, and only small amounts of the other light isotopes, $^2$H, $^3$H, $^3$He, $^7$Li, and $^7$Be, are produced.

The Coulomb barrier shuts off the nuclear reactions before there is time for heavier nuclei ($A > 8$) to form. One gets a fairly good approximation for the $^4$He production by assuming instantaneous nucleosynthesis at $T = T_{ns} \sim 1.1T_d \sim 70$ keV, with all neutrons ending up in $^4$He, so that

$$X_4 \approx 2X_n(T_{ns}).$$

(19)

After electron annihilation ($T \ll m_e = 0.511$ MeV) the time-temperature relation is

$$t = 0.301g^*\frac{m_{Pl}^3}{T^2},$$

(20)

where $g^* = 3.363$. Since most of the time in the temperature interval $T = 0.8$ MeV–0.07 MeV is spent at the lower part of this temperature range, this formula gives a good approximation for the time

$$t_{ns} - t_1 = 266.5 \text{ s} \quad \text{(in reality 264.3 s).}$$

Thus we get for the final $^4$He abundance

$$X_4 = 2X_n(t_1)e^{-(t_{ns} - t_1)/\tau_n} = 24.5\%.$$  

(21)

Accurate numerical calculations, using the reaction rates of the relevant weak and strong reaction rates give $X_4 = 21$–26\% (for $\eta = 10^{-10} - 10^{-9}$).

As a calculation of the helium abundance $X_4$ the preceding calculation is of course a cheat, since we have used the results of those accurate numerical calculations to infer that we need to use $T = 0.8$ MeV as the neutrino decoupling temperature, and $T_{ns} = 1.1T_d$ as the “instantaneous nucleosynthesis” temperature, to best approximate the correct behavior. However, it gives us a quantitative description of what is going on, and an understanding of how the helium yield depends on various things.

**Exercise**: Using the preceding calculation, find the dependence of $X_4$ on $\eta$, i.e., calculate $dX_4/d\eta$.

### 5.5 Why so late?

Let us return to the question, why the temperature has to fall so much below the binding energy before the equilibrium abundances become large. From the energetics one might conclude that when typical kinetic energies, $\langle E_k \rangle \approx \frac{3}{2}T$ for nuclei and $\langle E \rangle \approx 2.7T$ for photons, are smaller than the binding energy, it would be easy to form nuclei but difficult to break them. Above we saw that the smallness of the factor $\epsilon \sim (T/m_N)^{3/2}/\eta$ is the reason why this is not so. Here $\eta \sim 10^{-9}$ and $(T/m_N)^{3/2} \sim 10^{-6}$ (for $T \sim 0.1$ MeV). The main culprit is thus the small baryon/photon ratio. Since there are $10^9$ photons for each baryon, there is a sufficient amount of photons who can disintegrate a nucleus in the high-energy tail of the photon distribution, even at rather low temperatures. One can also express this result in terms of entropy. A high photon/baryon ratio corresponds to a high entropy per baryon. High entropy favors free nucleons.

### 5.6 The most important reactions

In reality, neither neutrino decoupling, nor nucleosynthesis, are instantaneous processes. Accurate results require a rather large numerical computation where one uses the cross sections of all the relevant weak and strong interactions. These cross sections are energy-dependent.
Integrating them over the energy and velocity distributions and multiplying with the relevant number densities leads to temperature-dependent reaction rates. The most important reactions are the weak $n \leftrightarrow p$ reactions (12) and the following strong reactions\(^4\) (see also Fig. 2):

\[
\begin{align*}
  p + n & \rightarrow 2H + \gamma \\
  2H + p & \rightarrow 3He + \gamma \\
  2H + 2H & \rightarrow 3H + p \\
  2H + 2H & \rightarrow 3He + n \\
  n + 3He & \rightarrow 3H + p \\
  p + 3H & \rightarrow 4He + \gamma \\
  2H + 3H & \rightarrow 4He + n \\
  2H + 3He & \rightarrow 4He + p \\
  4He + 3He & \rightarrow 7Be + \gamma \\
  4He + 3H & \rightarrow 7Li + \gamma \\
  7Be + n & \rightarrow 7Li + p \\
  7Li + p & \rightarrow 4He + 4He \\
\end{align*}
\]

The cross sections of these strong reactions can’t be calculated from first principles, i.e., from QCD, since QCD is too difficult. Instead one uses cross sections measured in laboratory. The cross sections of the weak reactions (12) are known theoretically (there is one parameter describing the strength of the weak interaction, which is determined experimentally, in practice by measuring the lifetime $\tau_n$ of free neutrons). The relevant reaction rates are now known sufficiently accurately, so that the nuclear abundances produced in BBN (for a given value of $\eta$) can be calculated with better accuracy than the present abundances can be measured from astronomical observations.

The reaction chain proceeds along stable and long-lived (compared to the nucleosynthesis timescale—minutes) isotopes towards larger mass numbers. At least one of the two incoming

\(^4\)The reaction chain that produces helium from hydrogen in BBN is not the same that occurs in stars. The conditions is stars are different: there are no free neutrons and the temperatures are lower, but the densities are higher and there is more time available. In addition, second generation stars contain heavier nuclei (C,N,O) which can act as catalysts in helium production. Some of the most important reaction chains in stars are ([2], p. 251):

1. The proton-proton chain

\[
\begin{align*}
  p + p & \rightarrow 2H + e^+ + \nu_e \\
  2H + p & \rightarrow 3He + \gamma \\
  3He + 3He & \rightarrow 4He + p + p, \\
\end{align*}
\]

2. and the CNO-chain

\[
\begin{align*}
  12C + p & \rightarrow 13N + \gamma \\
  13N & \rightarrow 13C + e^+ + \nu_e \\
  13C + p & \rightarrow 14N + \gamma \\
  14N + p & \rightarrow 15O + \gamma \\
  15O & \rightarrow 15N + e^+ + \nu_e \\
  15N + p & \rightarrow 12C + 4He. \\
\end{align*}
\]

The cross section of the direct reaction $d+d \rightarrow 4He + \gamma$ is small (i.e., the $^3H + p$ and $^3He + n$ channels dominate $d+d \rightarrow$), and it is not important in either context.

The triple-$\alpha$ reaction $^4He + ^4He + ^4He \rightarrow 12C$, responsible for carbon production in stars, is also not important during big bang, since the density is not sufficiently high for three-particle reactions to occur (the three $^4He$ nuclei would need to come within the range of the strong interaction within the lifetime of the intermediate state, $^8$Be, $2.6\times10^{-16}$ s). (Exercise: calculate the number and mass density of nucleons at $T = 1$ MeV.)
5 BIG BANG NUCLEOSYNTHESIS

The 12 most important nuclear reactions in big bang nucleosynthesis.

Figure 2: The 12 most important nuclear reactions in big bang nucleosynthesis.

nuclei must be an isotope which is abundant during nucleosynthesis, i.e., n, p, $^2$H or $^4$He. The mass numbers $A = 5$ and $A = 8$ form bottlenecks, since they have no stable or long-lived isotopes. These bottlenecks cannot be crossed with n or p. The $A = 5$ bottleneck is crossed with the reactions $^4$He+$^3$He and $^4$He+$^3$H, which form a small number of $^7$Be and $^7$Li. Their abundances remain so small that we can ignore the reactions (e.g., $^7$Be + $^4$He $\rightarrow ^{11}$C + $\gamma$ and $^7$Li + $^4$He $\rightarrow ^{11}$B + $\gamma$) which cross the $A = 8$ bottleneck. Numerical calculations also show that the production of the other stable lithium isotope, $^6$Li is several orders of magnitude smaller than that of $^7$Li.

Thus BBN produces the isotopes $^2$H, $^3$H, $^3$He, $^4$He, $^7$Li and $^7$Be. Of these, $^3$H (half life 12.3 a) and $^7$Be (53 d) are unstable and decay after nucleosynthesis into $^3$He and $^7$Li. ($^7$Be actually becomes $^7$Li through electron capture $^7$Be + e$^-$ $\rightarrow ^7$Li + $\nu_e$.)

In the end BBN has produced cosmologically significant (compared to present abundances) amounts of the four isotopes, $^2$H, $^3$He, $^4$He and $^7$Li (the fifth isotope $^1$H = p we had already before BBN). Their production in the BBN can be calculated, and there is only one free parameter, the baryon/photon ratio

$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{\Omega_b \rho_{cr0}}{m_N n_\gamma} = \frac{\Omega_b}{m_n n_\gamma} \frac{3H_0^2}{8\pi G}$$

$$= 274 \times 10^{-10} \omega_b = 1.46 \times 10^{18} \left(\frac{\rho_{b0}}{\text{kgm}^{-3}}\right).$$

Here $\rho_{b0}$ is the average density of ordinary, or baryonic, matter today, $\Omega_b \equiv \rho_{b0}/\rho_{cr0}$ is the baryon density parameter, and $\omega_b \equiv \Omega_b h^2$. 
Figure 3: The time evolution of the n, $^2$H (written as d) and $^4$He abundances during BBN. Notice how the final $^4$He abundance is determined by the n abundance before nuclear reactions begin. Only a small part of these neutrons decay or end up in other nuclei. Before becoming $^4$He, all neutrons pass through $^2$H. To improve the visibility of the deuterium curve, we have plotted it also as multiplied by a factor of 50. This figure is for $\eta = 6 \times 10^{-10}$. The time at $T = (90, 80, 70, 60)$ keV is $(152, 199, 266, 367)$ s. Thus the action peaks at about $t = 4$ min. The other abundances (except p) remain so low, that to see them the figure must be redrawn in logarithmic scale (see Fig. 4). From [3].

Figure 4: Time evolution of the abundances of the light isotopes during BBN. From http://www.astro.ucla.edu/~wright/BBNS.html

5.7 BBN as a function of time

Let us follow nucleosynthesis as a function of time (or decreasing temperature). See Figs. 3 and 4. $^2$H and $^3$H are intermediate states, through which the reactions proceed towards $^4$He. Therefore their abundance first rises, is highest at the time when $^4$He production is fastest, and then falls as the baryonic matter ends in $^4$He. $^3$He is also an intermediate state, but the main channel from $^3$He to $^4$He is via $^3$He+n→$^3$H+p, which is extinguished early as the free neutrons are used up. Therefore the abundance of $^3$He does not fall the same way as $^2$H and $^3$H. The abundance of $^7$Li also rises at first and then falls via $^7$Li+p→$^4$He+$^3$He. Since $^4$He has a higher binding energy per nucleon, $B/A$, than $^7$Li and $^7$Be have, the nucleons in them also want to return into $^4$He. This does not happen to $^7$Be, however, since, just like for $^3$He, the free neutrons needed for the reaction $^7$Be+n→$^3$He+$^4$He have almost disappeared near the end.
5.8 Primordial abundances as a function of the baryon-to-photon ratio

Let us then consider BBN as a function of $\eta$ (see Fig. 5). The larger is $\eta$, the higher is the number density of nucleons. The reaction rates are faster and the nucleosynthesis can proceed further. This means that a smaller fraction of “intermediate nuclei”, $^2\text{H}$, $^3\text{He}$, and $^7\text{Li}$ are left over—the burning of nuclear matter into $^4\text{He}$ is “cleaner”. Also the $^3\text{He}$ production falls with increasing $\eta$. However, $^7\text{Be}$ production increases with $\eta$. In the figure we have plotted the final BBN yields, so that $^3\text{He}$ is the sum of $^3\text{He}$ and $^3\text{H}$, and $^7\text{Li}$ is the sum of $^7\text{Li}$ and $^7\text{Be}$. The complicated shape of the $^7\text{Li}(\eta)$ curve is due to these two contributions: 1) For small $\eta$ we get lots of “direct” $^7\text{Li}$, whereas 2) for large $\eta$ there is very little “direct” $^7\text{Li}$ left, but a lot of $^7\text{Be}$ is produced. In the middle, at $\eta \sim 3 \times 10^{-10}$, there is a minimum of $^7\text{Li}$ production where neither way is very effective.

The $^4\text{He}$ production increases with $\eta$, since with higher density nucleosynthesis begins earlier when there are more neutrons left.
Figure 6: Determining the baryon/photon ratio $\eta$ by comparing BBN predictions to observations. The width of the bands around the the curves represents the uncertainty in BBN prediction due to uncertainty about the reaction rates. The vertical extent of the yellow boxes represent the estimate of the primordial abundance from observations and the horizontal extent the resulting range in $\eta$ to agree with BBN. Note the small deuterium box. The only observational data on $^3$He is from our own galaxy, and since $^3$He is both produced and destroyed in chemical evolution, we cannot infer the primordial abundance from them. From the review by Fields, Molaro, and Sarkar in [1].
5.9 Comparison with observations

The abundances of the various isotopes calculated from BBN can be compared to the observed abundances of these elements. This is one of the most important tests of the big bang theory. A good agreement is obtained for $\eta$ in the range $\eta = 5.8-6.6 \times 10^{-10}$. This was the best method to estimate the amount of ordinary matter in the universe, until the advent of accurate CMB data, first from the WMAP satellite starting in 2003 and then from the Planck satellite data starting in 2013.\(^5\)

The comparison of calculated abundances with observed abundances is complicated due to chemical evolution. The abundances produced in BBN are the primordial abundances of these isotopes. The first stars form with this composition. In stars, further fusion reactions take place and the composition of the star changes with time. Towards the end of its lifetime, the star ejects its outer parts into interstellar space, and this processed material mixes with primordial material. From this mixed material later generation stars form, and so on.

The observations of present abundances are based on spectra of interstellar clouds and stellar surfaces. To obtain the primordial abundances from the present abundances the effect of chemical evolution has to be estimated. Since $^2$H is so fragile (its binding energy is so low), there is hardly any $^2$H production in stars, rather any pre-existing $^2$H is destroyed early on in stars. Therefore any interstellar $^2$H is primordial. The smaller the fraction of processed material in an interstellar cloud, the higher its $^2$H abundance should be. Thus all observed $^2$H abundances are lower limits to the primordial $^2$H abundance.\(^6\) Conversely, stellar production increases the $^4$He abundance. Thus all $^4$He observations are upper limits to the primordial $^4$He. Moreover, stellar processing produces heavier elements, e.g., C, N, O, which are not produced in the BBN. Their abundance varies a lot from place to place, giving a measure of how much chemical evolution has happened in various parts of the universe. Plotting $^4$He vs. these heavier elements one can extrapolate the $^4$He abundance to zero chemical evolution to obtain the primordial abundance.

Since $^3$He and $^7$Li are both produced and destroyed in stellar processing, it is more difficult to make estimates of their primordial abundances based on observed present abundances.

Quantitatively, one can note two clear signatures of big bang in the present universe:

1. All stars and gas clouds observed contain at least 23% $^4$He. If all $^4$He had been produced in stars, we would see similar variations in the $^4$He abundance as we see, e.g., for C, N, and O, with some regions containing just a few % or even less $^4$He. This universal minimum amount of $^4$He must signify a primordial abundance produced when matter in the universe was uniform.

2. The existence of significant amounts of $^2$H in the universe is a sign of BBN, since there are no other known astrophysical sources of large amounts of $^2$H.

Quantitatively, the observed abundances of all the BBN isotopes, $^2$H, $^3$He, $^4$He and $^7$Li point towards the range $\eta = 1.5-7 \times 10^{-10}$. See Fig. 6. Since $^2$H has the steepest dependence on $\eta$, it can determine $\eta$ the most accurately. The best $^2$H observations for this purpose are from the absorption spectra of distant (high-z) quasars. This absorption is due to gas clouds that lie on the line-of-sight between us and the quasar. Some of these clouds lie also at a high redshift. Thus we observe them as they were when the universe was rather young, and therefore little chemical evolution had yet taken place. These measurements point towards the higher end of

\(^5\)Many cosmological parameters can be estimated from the CMB anisotropy, as will be discussed in Cosmology II. The Planck estimate\(^5\) is $\omega_b = 0.0224 \pm 0.0001$, or $\eta = (6.14 \pm 0.03) \times 10^{-10}$.

\(^6\)This does not apply to sites which have been enriched in $^2$H due to a separation of $^2$H from $^1$H. Deuterium binds into molecules more easily than ordinary hydrogen. Since deuterium is heavier than ordinary hydrogen, deuterium and deuterated molecules have lower thermal velocities and do not escape from gravity as easily. Thus planets tend to have high deuterium-to-hydrogen ratios.
the above range, to $\eta = 5.8-6.6 \times 10^{-10}$. Constraints from $^3$He and $^4$He are less accurate but consistent with this range. The estimates based on $^7$Li abundances in the surfaces of a certain class of old Population II stars, which have been thought to retain the primordial abundance, give lower values $\eta = 1.5-4.5 \times 10^{-10}$. This is known as the “Lithium problem”. It is usually assumed that we do not understand well enough of the physics of the stars in question, and the range $\eta = 5.8-6.6 \times 10^{-10}$ (at 95% confidence level) is taken as the BBN value for the baryon-to-photon ratio.[1] (This is also consistent with the CMB results.)

The wider range $\eta = 1.5-7 \times 10^{-10}$ corresponds to $\omega_b = \Omega_b h^2 = 3.65 \times 10^7 \eta = 0.0055-0.026$. With $h = 0.7 \pm 0.07$, this gives $\Omega_b = 0.009-0.07$ for a conservative range of the baryonic density parameter. With $\eta = 5.8-6.6 \times 10^{-10}$ and $h = 0.7 \pm 0.07$, the BBN result for the baryonic density parameter is

$$\Omega_b = 0.036-0.061.$$  \hspace{1cm} (23)

This is less than cosmological estimates for $\Omega_m$, which are around 0.3. Therefore not all matter can be baryonic. In fact, most of the matter in the universe appears to be nonbaryonic dark matter. This is discussed in Chapter 6.

### 5.10 BBN as a probe of the early universe

BBN is the earliest event in the history of the universe from which we have quantitative evidence in the form of numbers (primordial abundances of $^2$H, $^3$He, $^4$He, $^7$Li) that we can calculate from known theory and compare to observations. It can be used to constrain many kinds of speculations about the early universe. For example, suppose there were additional species of particles that were relativistic at BBN time (this is called *dark radiation*). This would increase $g_*$ and speed up the timescale (20), leading to more primordial $^4$He and a higher primordial abundance of the intermediate isotopes $^2$H and $^3$He. Most such modifications of the standard picture will ruin the agreement between theory and observations. Thus we can say that we know well the history of the universe since the beginning of the BBN (from $T \sim 1$ MeV and $t \sim 1$ s), but before that there is much more room for speculation.

### References

[1] Particle Data Group, Chinese Physics C 40, 100001 (2016)


