2. Universe with homogeneous, massless scalar field

\[ V(\phi) = 0 = \text{massless free scalar field} \]

\[ V = \text{massless inflation (field)} \]

\[ s = \frac{1}{2} \phi^2 + s \text{ (for non-homog. field)} \]

Assume homogeneous (\( \rho = \rho_0 \)), spatially flat universe.

Friedmann for flat universe with only inflation field is:

\[ H^2 = \frac{8\pi G}{3} \rho = \frac{1}{3} \rho = \frac{1}{3} \rho V + V(\phi) \]

\[ H = \frac{a}{a V} \rho \]

From energy continuity follows that:

\[ s = -3H \dot{\phi} \]

\[ s = \frac{1}{2} \phi^2 \]

\[ P = \frac{1}{2} \rho - V \]

\[ \ddot{s} + \frac{3}{V} \dot{s} = 0 \text{ (Field equation for } \phi \text{ field)} \]

For an \( \phi \) sub, we could try:

\[ \phi = \frac{e^x}{x} \]

\[ \ddot{\phi} = \frac{e^x}{x} \frac{e^x}{x} \]

\[ \dot{\phi} = \frac{e^x}{x} \frac{e^x}{x} \]

For a \( \phi \) sub:

\[ H = \frac{a}{a V} \rho = \frac{1}{2} M_p \phi^2 \]

\[ \phi = \frac{e^x}{x} \frac{e^x}{x} \]

\[ \rho = \frac{1}{2} M_p \phi^2 \frac{e^x}{x} \]

\[ \frac{\rho}{\rho} = \frac{1}{e^{2x}} \frac{e^x}{x} \]

The curvature term can be written as:

\[ H^2 = \frac{1}{2} M_p \phi^2 \frac{e^x}{x} \]

For matter-dom universe \( \phi, \frac{1}{2} = a \)

\[ H = \frac{a}{a \phi} \]

For universe dominated by massless inflation:

\[ a = \phi \]

So the magnitude of the curvature term behaves as \( \phi^2 \) in a universe dominated by field \( \phi \).

A universe dominated by massless scalar field becomes curvature dominated.

**Conclusion:** We might think of quasie"on dom by inflation field to decrease the curvature term as a function of time.

Contradiction here, so massless scalar field by itself is not a good candidate for inflation.