The density perturbations in the matter-dominated universe, for short-wavelength modes k \gg k_{\text{eq}}

Satisfy:

\[ \frac{\dot{\delta}}{a} + H \delta + \frac{k^2}{a^2} \delta = 0 \]

\[ \frac{\dot{\delta}}{a} + \frac{1}{a} \frac{\delta}{dt} \frac{\text{d}a}{\text{d}t} = \frac{k^2}{a^2} \delta \]

\[ \frac{\ddot{\delta}}{a^2} - \frac{2}{a} \frac{\dot{\delta}}{a} = -\frac{k^2}{a^2} \delta \]

\[ \frac{\ddot{\delta}}{a^2} + \frac{2}{a} \frac{\dot{\delta}}{a} = \frac{1}{a^2} \left( \frac{\text{d}a}{\text{d}t} \right)^2 - \frac{k^2}{a^2} \delta \]

\[ \Delta \delta + \frac{k^2}{a^2} \delta = 0 \]

For matter-dominated universe

H = \frac{\dot{a}}{a} + H_0 \delta(x, t) = H_{\text{eq}} V^{-1/2}(x) \sin \theta \]

\[ \Delta \delta = \left( \frac{\text{d}a}{\text{d}t} \right)^2 - \frac{k^2}{a^2} \delta = \frac{a}{a} \left( \frac{\text{d}a}{\text{d}t} \right)^2 - \frac{k^2}{a^2} \delta \]

Spherical Bessel functions satisfy:

\[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \left( \frac{\sin^2 \theta}{r^2} - \frac{\beta^2}{r^2} \right) \psi = 0 \]

Spherical Bessel differential equation.

Now \( r = 0 \), \( \alpha = (ck)^2 \), solutions are spherical Bessel functions of order 0.

\[ \delta_k(r) = A J_0(ckr) + B Y_0(ckr) \]

Amplitude

\[ \delta_k(r) = \frac{2}{ckr} \left[ A \sin(ckr) - B \cos(ckr) \right] \]

\[ \Delta \delta_k = \frac{2}{ckr} \left[ A \sin(ckr) - B \cos(ckr) \right] \]

Amplitude determined from initial conditions (haven't been set here).

\[ \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} \right) \psi + \frac{\beta^2}{r^2} \psi = 0 \]

Amplitude decreases as a function of time.

\[ \text{Frequency} \quad \frac{\text{d}^{2}\psi}{\text{d}t^{2}} + \frac{k}{a} \frac{\text{d}\psi}{\text{d}t} + \frac{k^2}{a^2} \psi = 0 \]

Amplitude of \( \psi \) decreases as \( a \) increases.

We have shown that scales much smaller than four lightyears.

Do not grow, but vanishes eventually.

In general, the \( \delta(x, t) \) for density perturbation:

\[ \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \right) + \frac{\beta^2}{a^2} \psi = 0 \]

Pressure terms dominate over pressure perturbations.

\[ \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \right) + \frac{\beta^2}{a^2} \psi = 0 \]

Amplitude decreases as a function of time.