1. The Einstein–deSitter model. Consider the spatially flat model with only matter, $\Omega_0 = \Omega_m = 1$.

(a) Calculate the scale factor $a(t)$, the age-redshift relation $t(z)$ and the angular diameter distance $d_A(z)$. (Express the age and distances in units of the Hubble time $H_0^{-1}$.)

(b) What is the horizon distance $d_{\text{hor}}$?

(c) What is the age of the universe (in years) today and at $z = 1090$ if $h = 0.7$?

(d) What is the angular diameter distance (in Mpc) to redshift $z = 1090$ if $h = 0.7$?

(e) The function $d_A(z)$ has a maximum. At what redshift is it?

2. Recession velocity. The recession velocity of a galaxy is the rate at which the proper distance $d(t)$ between us and the galaxy increases,

$$v_{\text{rec}} = \frac{dd(t)}{dt} = \dot{d}.$$  

For a given galaxy with redshift $z$ we may ask what is its recession velocity today, $v_{\text{rec}}(z, t_0)$, or what it was when the light left the galaxy, $v_{\text{rec}}(z, t(z))$. Give these two relations $v_{\text{rec}}(z)$ for the Einstein–de Sitter model. For what redshifts $z$ do they equal the speed of light? How far (in units of Hubble distance $H_0^{-1}$) are these galaxies (the ones with $v_{\text{rec}}(z, t_0) = 1$ and $v_{\text{rec}}(z, t(z)) = 1$) today?

3. Can you see around the closed universe? How long does it take for light to travel the full circle around the closed universe ($\Omega_m > 1, \Omega_\Lambda = 0$)? (Consider a light ray which originated at $t = 0$). Can you see your back? When do you see the entire universe?

4. \(\Lambda\)CDM model. Consider the flat, $\Omega_0 = \Omega_m + \Omega_\Lambda = 1$, FRW model with $H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$.

(a) Find the age of the universe today and at redshift $z = 1090$.

(b) When (give both $t$ and $z$) was the matter density equal to the vacuum energy density?

(c) In this model the expansion rate has an inflection point, where $a = 0$, and the expansion begins to accelerate. When did this happen?

Hint: Use substitution $x^{3/2} = b \sinh \phi$ for the integral

$$\int \frac{x^{1/2}dx}{\sqrt{b^2 + x^3}}$$