Due on Monday, November 5th, by 14.15 o’clock.

1. **Oldness problem.** Assume $\Omega_k(T = 100 \text{ keV}) = 0.1$. Include just curvature and radiation (with $g_* = 3.36$) in the Friedmann equation. How long does it take for the universe to cool to $T = 2.7 \text{ K}$? What is the curvature radius then? Why would inclusion of matter (with, say $\eta = 6 \times 10^{-10}$, and $\rho_m = 6 \rho_b$) not change the answers?

2. **Universe with homogeneous massless scalar field.** A massless free scalar field is one for which the potential is identically zero. Find the general solutions, i.e., give $\varphi(t)$ and $a(t)$, for homogeneous, spatially flat cosmologies containing such a scalar field (and no other matter). Choose the origin of time so that $H \to \infty$ when $t \to 0$ (as usual). Does such a universe become curvature dominated more or less easily than a matter-dominated universe?

3. **Power-law inflation.** Assume the field potential has the form

$$V(\varphi) = V_0 \exp \left(-\sqrt{\frac{2}{p} \frac{\varphi}{M_{\text{Pl}}}}\right), \quad p > 1.$$ 

Show that

$$a \propto t^p$$

$$\varphi = \sqrt{2p M_{\text{Pl}} \ln \left(\frac{V_0}{p(3p-1) M_{\text{Pl}}} t\right)}$$

is a solution to the field and Friedmann equations

$$\ddot{\varphi} + 3H \dot{\varphi} = -V'(\varphi)$$

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} \left[\frac{1}{2} \dot{\varphi}^2 + V(\varphi)\right].$$

4. **Slow-roll parameters.** Demonstrate that the two conditions,

$$\varepsilon(\varphi) \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V}\right)^2 \ll 1 \quad \text{and} \quad |\eta(\varphi)| \equiv \left|\frac{M_{\text{Pl}}^2 V''}{V}\right| \ll 1,$$

are necessary conditions for the slow-roll approximation to be valid. Why are these conditions not sufficient?