Production of $^4$He in Big Bang Nucleosynthesis

- Assume a single model where neutrinos decouple instantaneously at $T_e = 0.9 \text{ MeV}$, (1)
- Nucleosynthesis happens instantaneously at $T_n = 0.06 \text{ MeV}$ (2)
- and $\mu_n \ll T$. (3)

The neutron and proton equilibrium abundances hold until $T = T_e$, and then are

$$n_n = \frac{g_n}{2\pi} \left(\frac{m_n T}{2\pi}\right)^{3/2} e^{-m_n/m_n}$$

$$n_p = \frac{g_p}{2\pi} \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-m_p/m_p}$$

Since $g_p = g_p$ and $m_n \approx m_p$, so that we can approximate them equal outside an exp,

$$\frac{n_n}{n_p} = e^{-(m_n-m_p)/T - (\mu_n-\mu_p)/T}$$

The reaction $n + e^+ \leftrightarrow p + \nu_e$ is in equilibrium until $T_e$, so $\mu_n-\mu_p = M_n-M_p$,

and both $\mu$ and $\nu_e \ll T$.

$$\Rightarrow \frac{n_n}{n_p} \approx e^{-Q/T}, \quad \text{where} \quad Q = m_n-m_p = 1.293 \text{ MeV}$$

$$\Rightarrow \text{The neutron mass fraction} \quad X_n = \frac{n_n}{n_n+n_p} = \frac{e^{-Q/T}}{1+e^{-Q/T}} = 0.1921 \quad \text{at } T_e$$

Neutrinos then decay with mean lifetime $\tau_n = 880 \text{ s}$

$$\Rightarrow X_n(t_{ns}) = X_n(t_d) = e^{-(t_{ns}-t_d)/\tau_n}$$

For the mass scale, assume $G_N = 3.363$ for the whole doc

$$\Rightarrow t = 0.301 m_e T^2 g_{1/2} = 0.301 \times 1.22 \times 10^{22} \left(\frac{T}{\text{MeV}}\right)^2 \frac{1}{\text{MeV}} = 1.317 \times (\frac{T}{\text{MeV}})^2$$

(used $1 \text{ MeV} = 1.52 \times 10^3 \text{ s}$)

$$\Rightarrow t_d = 1.626 \text{ s} \quad \text{and} \quad t_{ns} = 365.8 \text{ s}$$

$$\Rightarrow X_n(t_{ns}) = 0.1921 \cdot e^{-(365.8 - 1.626)/880 \text{ s}} = 0.127$$

These neutrons combine with the same number of protons to form the nuclei

with $X_4 = 2X_n(t_{ns}) = 0.254 \approx 25.4\%$