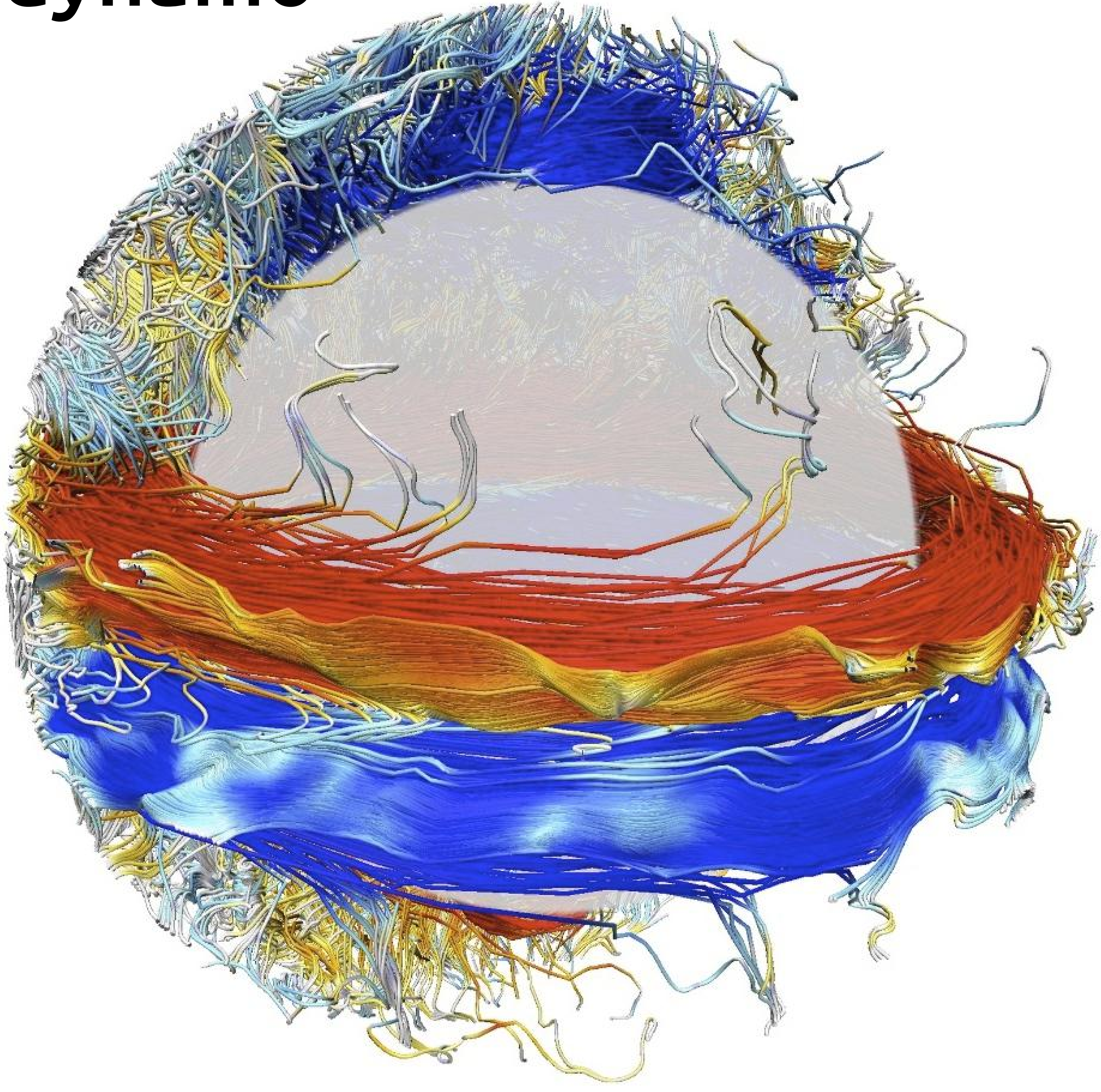
Solar dynamo



Origin of solar magnetic field

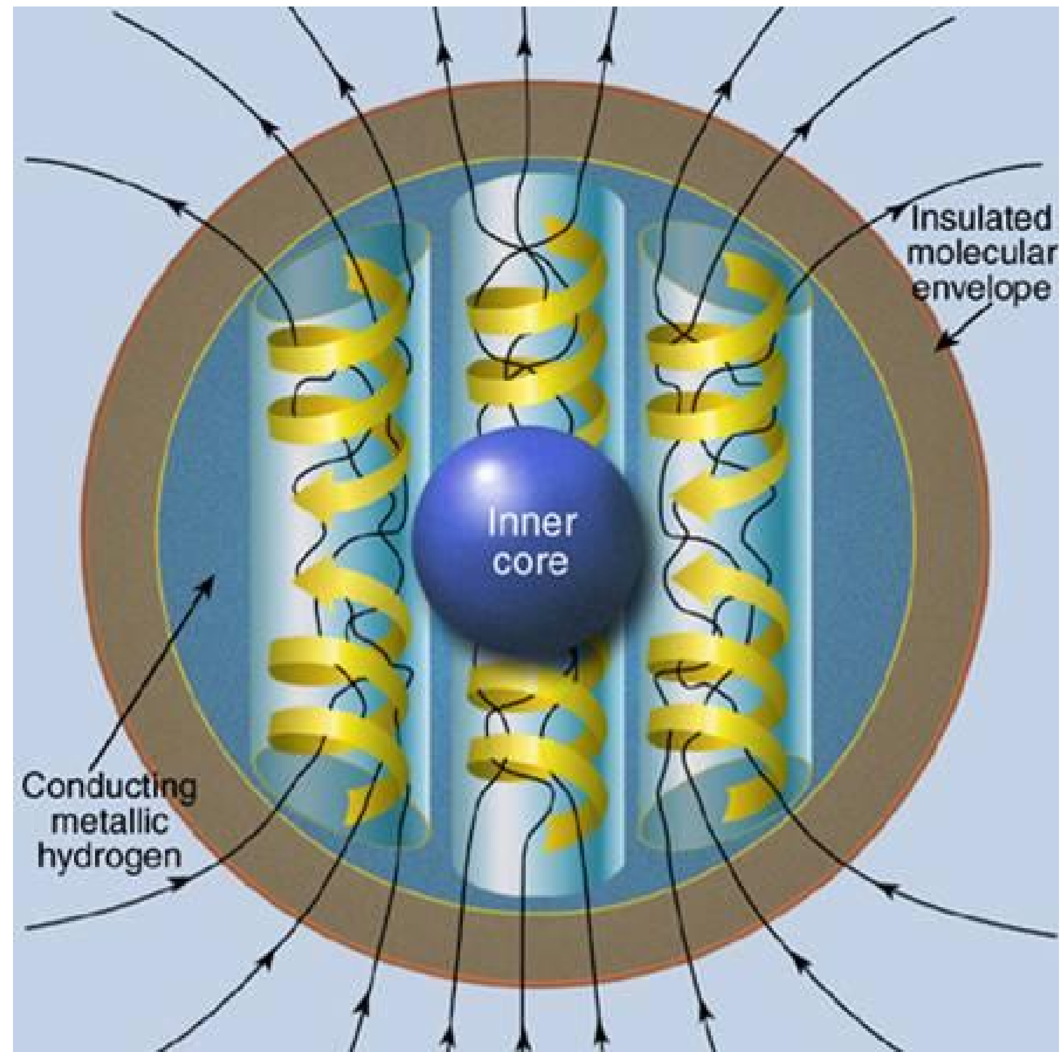
- Remnant magnetic field
  - Compressed magnetic field from initial cloud
  - Magnetic field is dissipated by Ohmic diffusion, but it still would exist nowadays
    → existence of magnetic field does not prove any dynamo

- Or magnetic field could be generated and sustained by some dynamo mechanism. Proved by:
  - 22-year solar cycle, polarity switches
  - Connection between differential rotation and sunspot migration towards equator

Earth dynamo

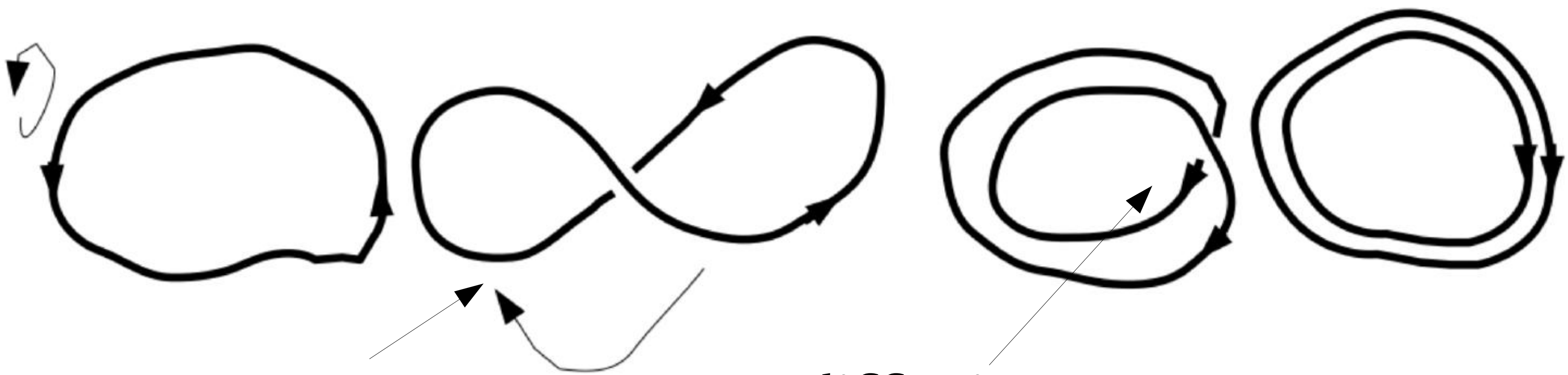
- In case of planets, including Earth, time of Ohmic diffusion is shorter
  → there should be generation of magnetic field via dynamo process
  → dynamo genertes magnetic flux

- On the Sun dynamo could generate OR sustain the pre-existing magnetic field



Idea of dynamo

Amplification of magnetic field via convection and diffusion

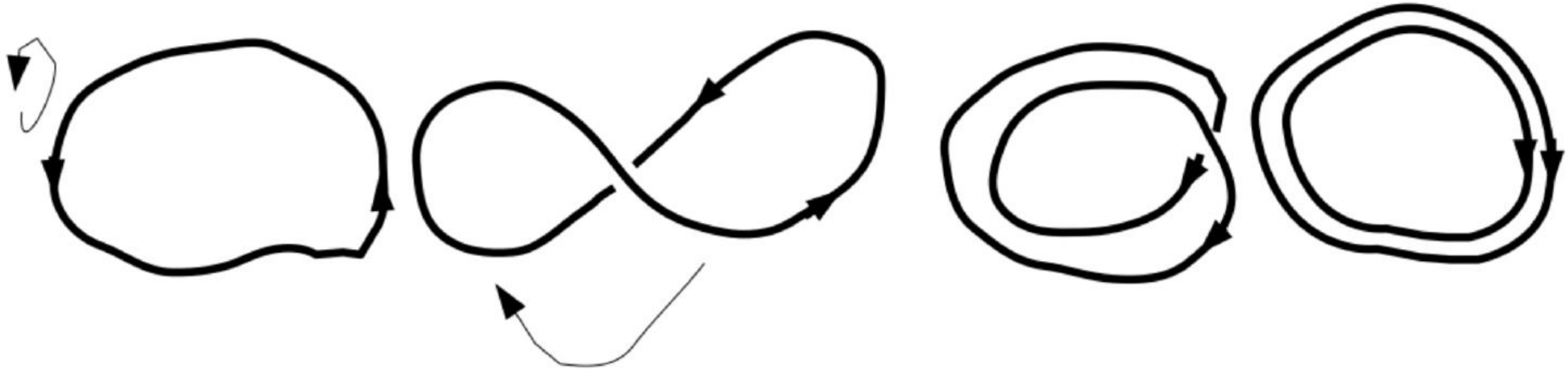


convection

diffusion

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Idea of dynamo



- Convection can amplify magnetic field locally
- Diffusion is required to generate new magnetic field
- Dynamo theory attempts to explain the balance between convection and diffusion required for efficient generation of magnetic flux

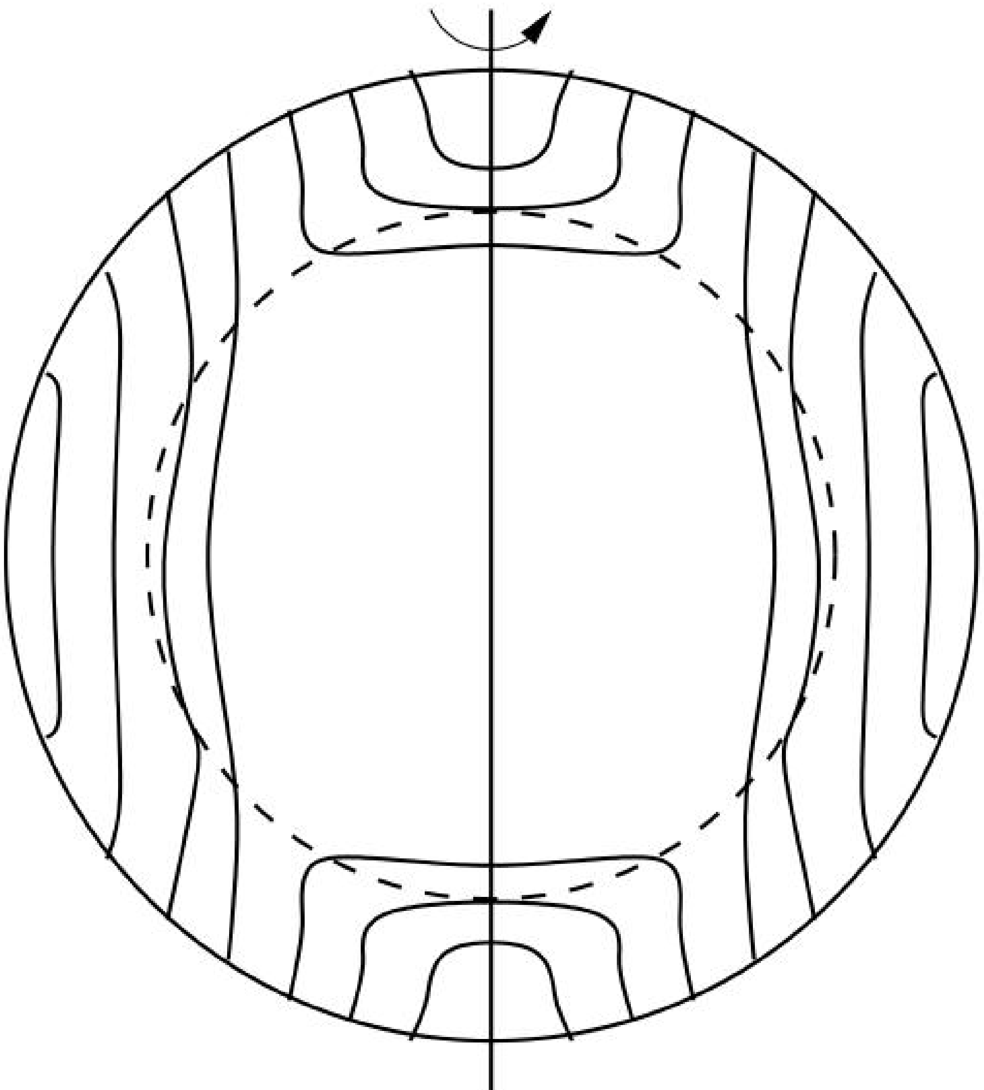
Kinematic & dynamic dynamos

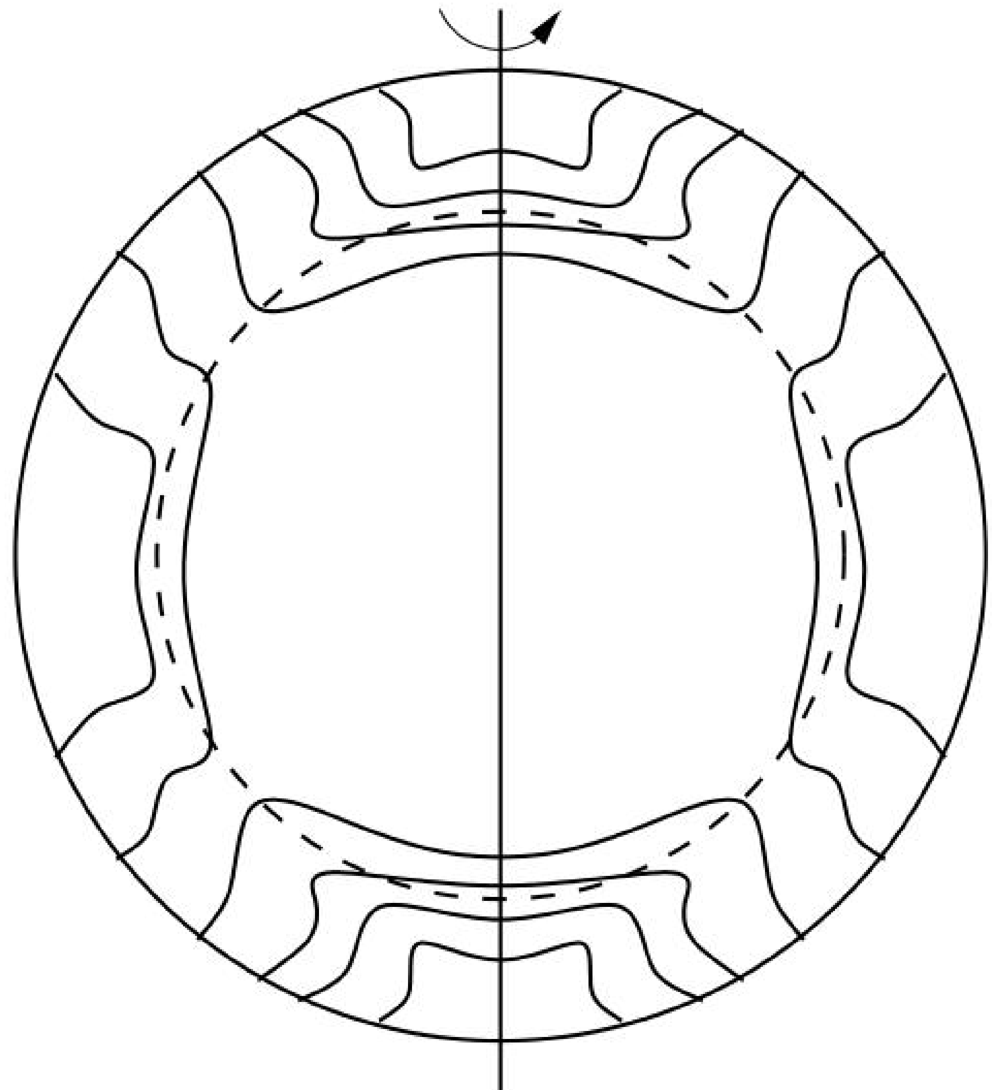
- Kinematic dynamo
  - Velocity field is given and does not depend on magnetic field
    → induction equation is linear

- Dynamic dynamo
  - Velocity field depends on magnetic field
    → non-linear induction equation

Kinematic & dynamic dynamos

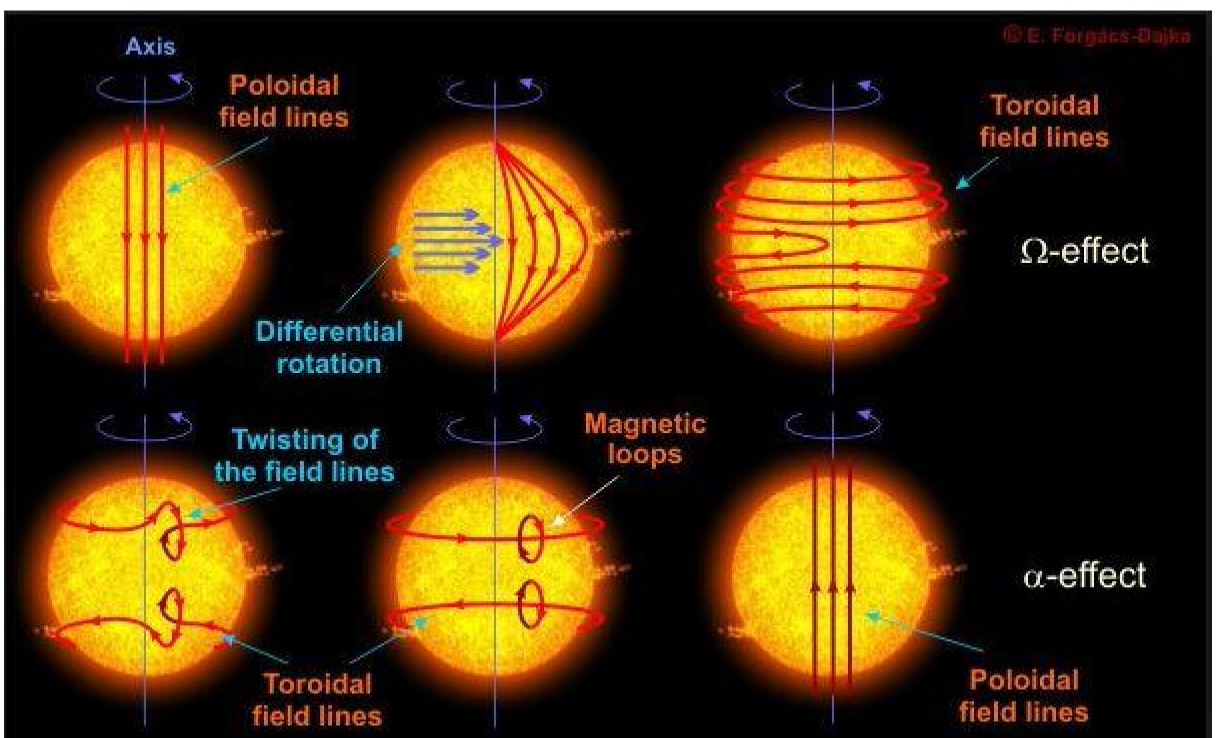
- Velocity shear in a narrow region
- Kinematic approach is not suitable



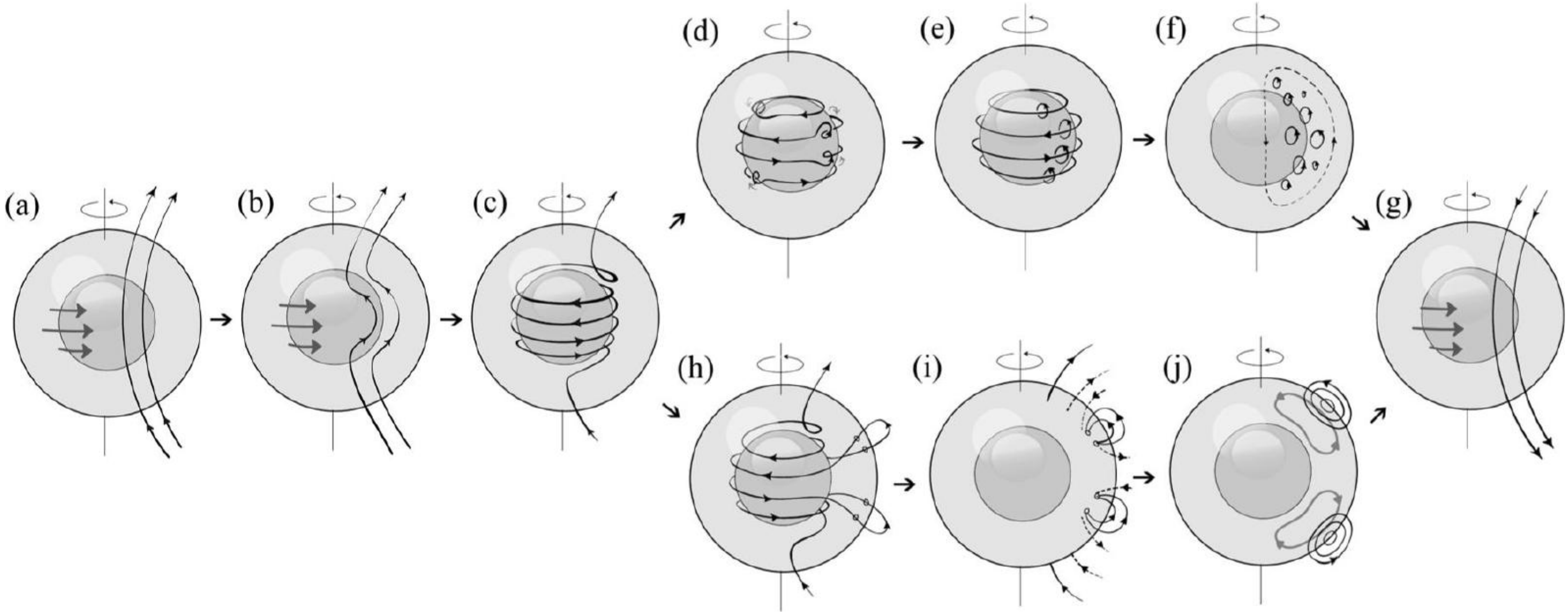


Parker's turbulent dynamo

- Toroidal magnetic field – circles around rotation axis
- Poloidal magnetic field – curves in meridional planes
- Differential rotation:
  toroidal → poloidal field
- Convection + Coriolis effect:
  poloidal → toroidal field



Babcock-Leighton dynamo



Mean-field approach

- Turbulent velocity field is too complicated → only statistical properties
- Differential rotation is important → mean only over longitudes

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$$
$$\mathbf{v} = \langle \mathbf{v} \rangle + \mathbf{u}$$

- Induction equation

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle + \mathcal{E} - \eta \nabla \times \langle \mathbf{B} \rangle)$$
$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\langle \mathbf{v} \rangle \times \mathbf{b} + \mathbf{u} \times \langle \mathbf{B} \rangle + \mathbf{G} - \eta \nabla \times \mathbf{b})$$

$$\boxed{\mathcal{E} = \langle \mathbf{u} \times \mathbf{b} \rangle} \qquad \mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle$$

- Ohm's law $\langle \mathbf{J} \rangle = \sigma(\langle \mathbf{E} \rangle + \langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle + \mathcal{E})$

Mean-field approach

- Turbulent motion → electric field → electric current → induced magnetic field

$$\boxed{\mathcal{E} \;=\; \langle \mathbf{u} \times \mathbf{b} \rangle} \quad \langle \mathbf{J} \rangle = \sigma(\langle \mathbf{E} \rangle + \langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle + \mathcal{E})$$

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} \;=\; \nabla \times (\langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle + \mathcal{E} - \eta \nabla \times \langle \mathbf{B} \rangle)$$

- Assuming linear relationship between b and <B> and incompressibility

$$\mathcal{E}_i = \alpha_{ij} \langle B_j \rangle + \beta_{ijk} \partial_k \langle B_j \rangle$$

Statistical properties of turbulent fields

Mean-field approach. α-effect

- First order smoothing approximation
  - Neglect $\mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle$
    - Valid for low Rm (in the Sun Rm is large)
  - Assume istotropic turbulence

$$\mathcal{E} = \alpha \langle \mathbf{B} \rangle - \beta \nabla \times \langle \mathbf{B} \rangle$$

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle + \alpha \langle \mathbf{B} \rangle - \eta_{\mathrm{t}} \nabla \times \langle \mathbf{B} \rangle)$$

$\eta_{\mathrm{t}} = \eta + \beta$ – total diffusivity

- In convection zone turbulent diffusivity dominates over regular diffusivity

α-effect: the rate of change of mean magnetic field depends on mean magnetic field itself

Mean-field approach. αω-effect

- Without differentialrotation α-effect can generate toroidal → poloidal and poloidal → toroidal
- If there is differential rotation poloidal → toroidal generation can go more effectively → ω-effect

$$\langle \mathbf{B} \rangle = \mathbf{B}_{\mathrm{p}} + \mathbf{B}_{\mathrm{t}}$$

$$\alpha(r, \pi - \theta) = -\alpha(r, \theta)$$

$$\mathbf{B}_{\mathrm{p}} = \nabla \times (0, 0, A(r, \theta, t))$$

$$\Omega(r, \pi - \theta) = \Omega(r, \theta)$$

$$\mathbf{B}_{\mathrm{t}} = (0, 0, B(r, \theta, t))$$

$$\langle \mathbf{v} \rangle = (0, 0, \Omega r \sin\theta)$$

$$\begin{aligned}
\frac{\partial A}{\partial t} &= \alpha B + \eta_t \nabla_1^2 A \\
\frac{\partial B}{\partial t} &= \frac{\partial \Omega}{\partial r} \frac{\partial}{\partial \theta}(A \sin\theta) - \frac{1}{r} \frac{\partial \Omega}{\partial \theta} \frac{\partial}{\partial r}(r A \sin\theta) - \frac{1}{r} \frac{\partial}{\partial r}\left[\alpha \frac{\partial}{\partial r}(r A)\right] \\
&\quad - \frac{1}{r^2} \frac{\partial}{\partial \theta}\left[\frac{\alpha}{\sin\theta} \frac{\partial}{\partial \theta}(A \sin\theta)\right] + \eta_{\mathrm{t}} \nabla_1^2 B \,,
\end{aligned}$$

Mean-field approach. αω-effect

$$\frac{\partial A}{\partial t} = \alpha B + \eta_t \nabla_1^2 A$$

$$\frac{\partial B}{\partial t} = \frac{\partial \Omega}{\partial r}\frac{\partial}{\partial \theta}(A\sin\theta) - \frac{1}{r}\frac{\partial \Omega}{\partial \theta}\frac{\partial}{\partial r}(rA\sin\theta) - \frac{1}{r}\frac{\partial}{\partial r}\left[\alpha\frac{\partial}{\partial r}(rA)\right] - \frac{1}{r^2}\frac{\partial}{\partial \theta}\left[\frac{\alpha}{\sin\theta}\frac{\partial}{\partial \theta}(A\sin\theta)\right] + \eta_{\mathrm{t}}\nabla_1^2 B\,,$$

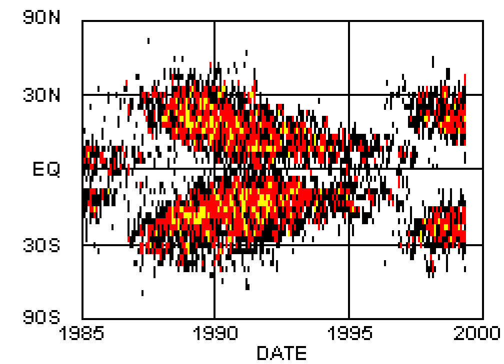
- If α=0 → poloidal field would decay
- If grad(Ω)=0 → α-effect would take over generation of toroidal field from poloidal

Sunspots migration

- Local region in northern hemisphere in cartsian coordinates, constant α, linear differential rotation

$$\mathbf{B}_\mathrm{p} = \left(0, 0, \frac{\partial A}{\partial x}\right)$$
$$\mathbf{B}_\mathrm{t} = (0, B, 0)$$

$$\dot{A} = \alpha B + \eta_\mathrm{t}\, \partial^2 A/\partial x^2$$
$$\dot{B} = \Omega_0\, \partial A/\partial x + \eta_\mathrm{t}\, \partial^2 B/\partial x^2$$



- Solution is a dynamo wave, propagates towards equator

$$(A, B) = (A_0, B_0)\exp[i(\omega t + kx)]$$

$$(i\omega + \eta_\mathrm{t} k^2)^2 = ik\Omega_0\alpha$$

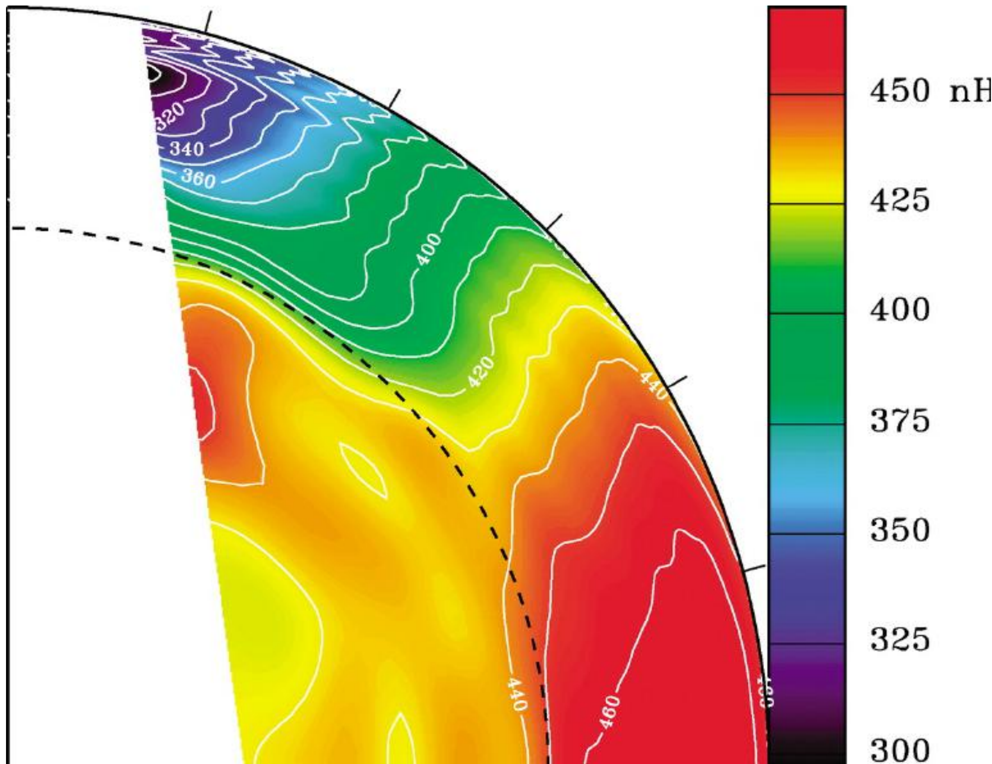
Mean-field approach. αω-effect

- Provides satisfactory description of the solar cycle
  - Minimum: no sunspots, large-scale poloidal field
  - Differential rotation destroys poloidal field and enhances toroidal field
  - Maximum: sunspots emerge in correspondence with Hale's polarity rules
  - Sunspots migrate from higher latitudes towards the equator
  - α-effect reorganizes poloidal field from toroidal
  - Next minimum: no sunspots, reversed polarity

Interface dynamo

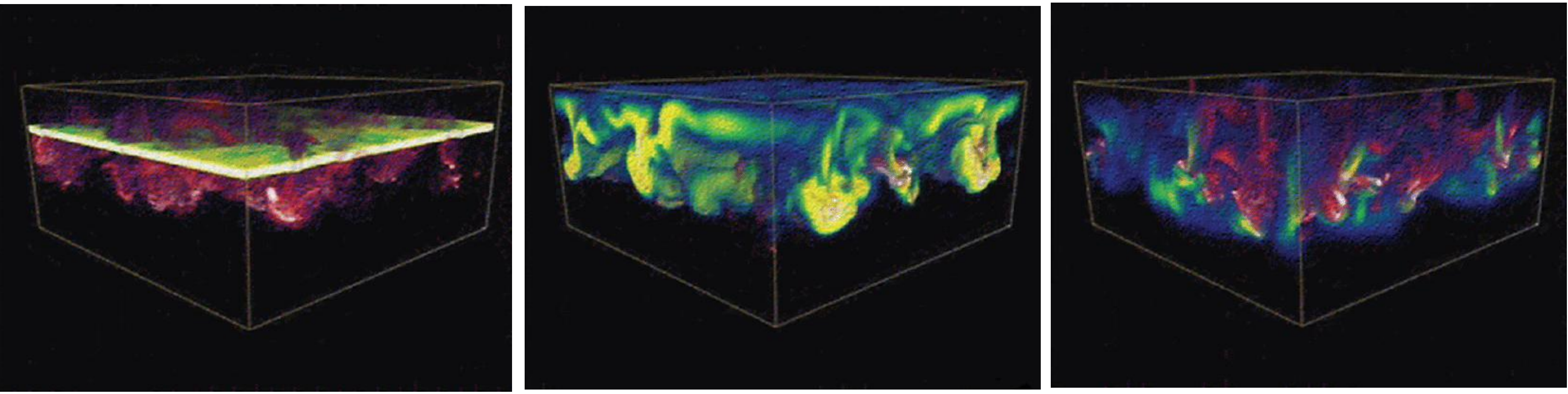
- α- and ω- effects are spatially separated:
  - ω-effect takes place closemto the tachocline (bottom of convection zone)
  - α-effect happens inside the convection zone or even in the higher layers of convection zone

  - Strongest velocity shear happens in the tachocline → stronger ω-effect
  - Toroidal field flows up from tachocline because of buoyancy



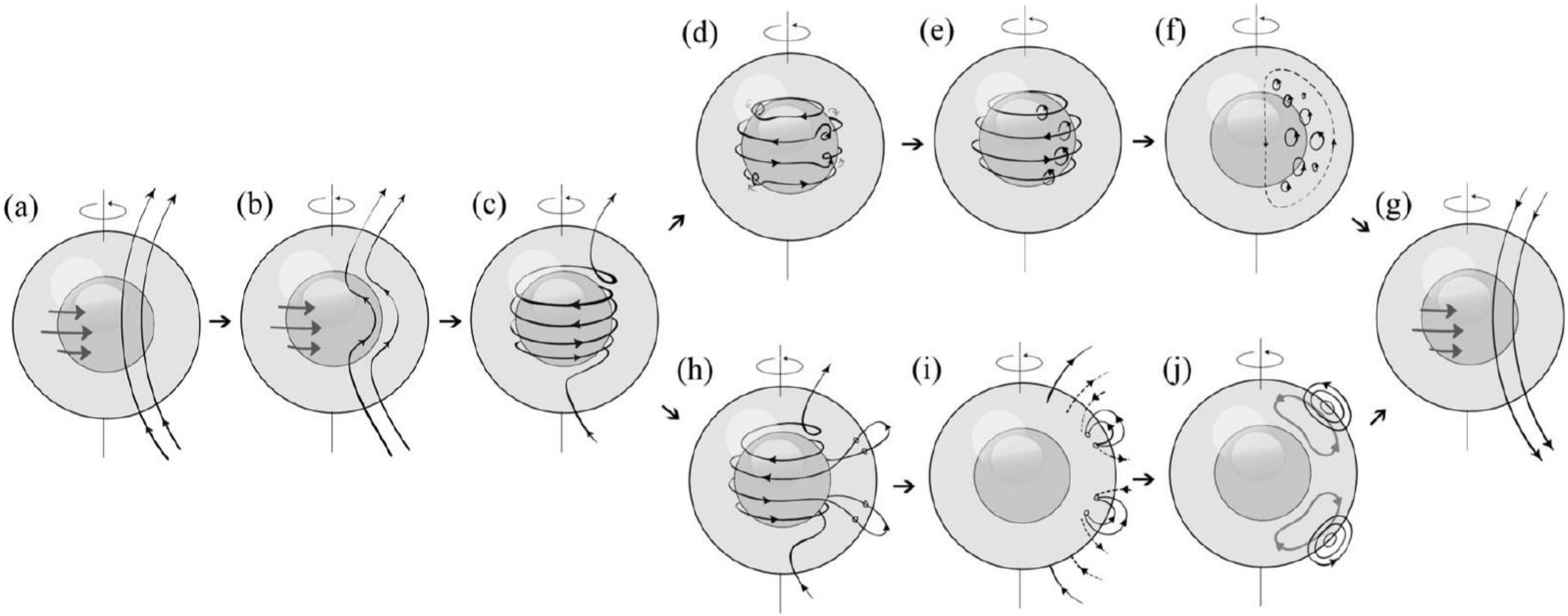
Magnetic pumping

Magnetic field dragged down by turbulent convection



- Magnetic pumping is the downwards expulsion of magnetic flux by turbulence. It can effectively return poloidal field to the tachocline.
- Turbulent convection zone acts as a filter → allows only the strongest field to rise to the surface and appear as active regions.

Location of α-effect



- Parker → α-effect inside the convection zone → problems with generating field of observed strength
- Babcock → α-effect on the surface → cannot work at times of grand minima
- α-effect close to the tachocline? Could be driven by buoyancy instabilities

Real dynamo is more complicated

- Solar cycle appears to have some long-period modulation: Maunder minimum, Dalton minimum → there is more to solar dynamo than a basic cycle